Corrigé du DM8
1FTP

Exercice 3 – chap3

$$Z(\omega) = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} \qquad \boxed{\omega > 0}$$

1) $$Z'(w) = \frac{\cancel{2}\left(Lw - \frac{1}{Cw}\right)\left(L + \frac{1}{Cw^2}\right)}{\cancel{2}\sqrt{R^2 + \left(Lw - \frac{1}{Cw}\right)^2}}$$

est du signe de $L\omega - \frac{1}{C\omega}$

$$Z'(w) \geq 0 \Leftrightarrow Lw - \frac{1}{Cw} \geq 0$$
$$\Leftrightarrow Lw \geq \frac{1}{Cw}$$
$$\Leftrightarrow LCw^2 \geq 1$$
$$\Leftrightarrow w^2 \geq \frac{1}{LC}$$
$$\Leftrightarrow \sqrt{w^2} \geq \sqrt{\frac{1}{LC}}$$
$$\Leftrightarrow |w| \geq \frac{1}{\sqrt{LC}}$$
$$\Leftrightarrow w \geq \frac{1}{\sqrt{LC}}$$

Tableau

w	0		$\frac{1}{\sqrt{LC}}$		$+\infty$
$Z'(w)$	‖	$-$	0	$+$	
Z	‖ $+\infty$	↘	$R = Z\left(\frac{1}{\sqrt{LC}}\right)$	↗	$+\infty$

$$Z\left(\frac{1}{\sqrt{LC}}\right) = \sqrt{R^2 + \underbrace{\left(\frac{L}{\sqrt{LC}} - \frac{1}{C\frac{1}{\sqrt{LC}}}\right)^2}_{=0}} = R$$

$$\frac{L}{\sqrt{LC}} = \frac{\sqrt{L}\sqrt{L}}{\sqrt{L}\sqrt{C}} = \frac{\sqrt{L}}{\sqrt{C}}$$

$$\frac{1}{C\frac{1}{\sqrt{LC}}} = \frac{\sqrt{LC}}{C} = \frac{\sqrt{L}\sqrt{C}}{\sqrt{C}\sqrt{C}} = \frac{\sqrt{L}}{\sqrt{C}}$$

② $I(\omega) = \frac{U}{Z(\omega)} = U \times \frac{1}{Z(\omega)}$ où U est une cte > 0

$I'(\omega) = \left(\frac{1}{V}\right)' = \frac{-U.Z'(\omega)}{Z(\omega)^2}$ est du signe opposé de $Z'(\omega)$

Tableau de Z :

ω	0		$\frac{1}{\sqrt{LC}}$		$+\infty$
$Z'(\omega)$	‖	$-$	0	$+$	
Z	‖ $+\infty$	↘	$R = Z\left(\frac{1}{\sqrt{LC}}\right)$	↗	$+\infty$

Tableau de I : $I(\omega) = \frac{U}{Z(\omega)}$

ω	0		$\frac{1}{\sqrt{LC}}$		$+\infty$
$I'(\omega)$	‖	$+$	0	$-$	
I	‖ 0	↗	$\frac{U}{R}$	↘	0

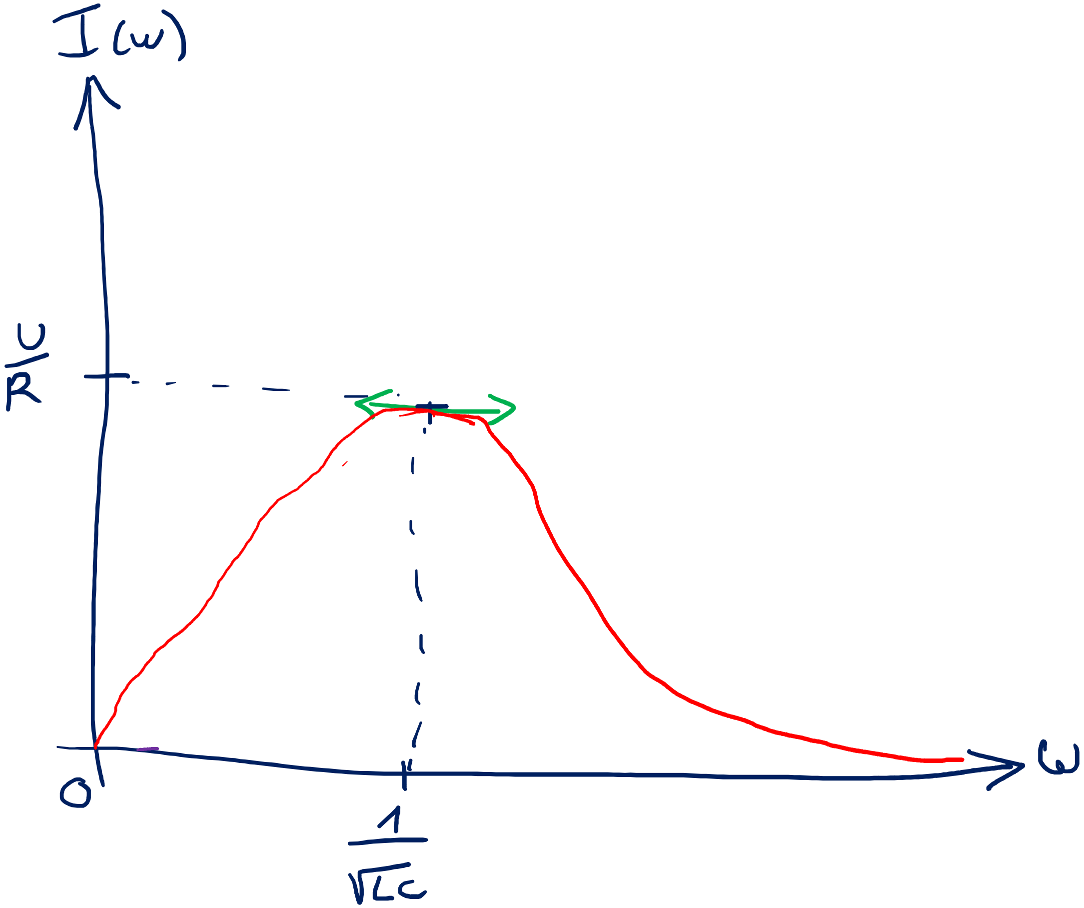

Tableau de I : $I(\omega) = \dfrac{U}{Z(\omega)}$

tangente horizontale

asymptote

ω	0		$\frac{1}{\sqrt{LC}}$		$+\infty$
$I'(\omega)$	‖	$+$	0	$-$	
	‖ 0	$\nearrow$	$\frac{U}{R}$	$\searrow$	0

Exercice 4 – chap3

ex4 $\quad P(R) = E^2 \cdot \dfrac{R}{(R+r)^2}$ $\qquad R > 0 \ ; \ r > 0$

$$P'(R) = \left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$= E^2 \cdot \frac{(R+r)^2 - R \cdot 2(R+r)}{(R+r)^4}$$

$$\Leftrightarrow p'(R) = E^2 \frac{(R+r)(R+r-2R)}{(R+r)^4}$$

$$p'(R) = E^2 \frac{r-R}{(R+r)^3}$$

$$P'(R) \geq 0 \Leftrightarrow (R+r)^2 - 2R \cdot (R+r) \geq 0$$

$$\Leftrightarrow R^2 + \cancel{2Rr} + r^2 - 2R^2 - \cancel{2Rr} \geq 0$$

$$\Leftrightarrow -R^2 + r^2 \geq 0$$

$$p'(R) \geq 0 \Leftrightarrow r - R \geq 0$$

$$\Leftrightarrow R \leq r$$

$$\Longleftrightarrow -R^2 + r^2 \geqslant 0$$

$$\Longleftrightarrow r^2 \geqslant R^2 \Longleftrightarrow r \geqslant R$$

$$P(R) = E^2 \frac{R}{(R+r)^2}$$

R	0		r		$+\infty$
$P'(R)$		$+$	0	$-$	
P	0	$\nearrow$	P_{max}	$\searrow$	0

$$P_{max}(R=r) = E^2 \cdot \frac{r}{(r+r)^2} = E^2 \cdot \frac{r}{(2r)^2} = E^2 \cdot \frac{r}{4r^2} = E \cdot \frac{1}{4r}$$

$$P_{max} = \frac{E^2}{4r}$$ est la puissance maximale.