

Corrigé du DM8

1FTP

Exercice 3 - chap3

$$Z(\omega) = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$\boxed{\omega > 0}$$

$$1) Z'(\omega) = \frac{2(L\omega - \frac{1}{C\omega})(L + \frac{1}{C\omega^2})}{2\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}}$$

et de la ligne de $L\omega - \frac{1}{C\omega}$

$$Z'(\omega) \geq 0 \Leftrightarrow L\omega - \frac{1}{C\omega} \geq 0$$

$$\Leftrightarrow L\omega \geq \frac{1}{C\omega}$$

$$\Leftrightarrow L C \omega^2 \geq 1$$

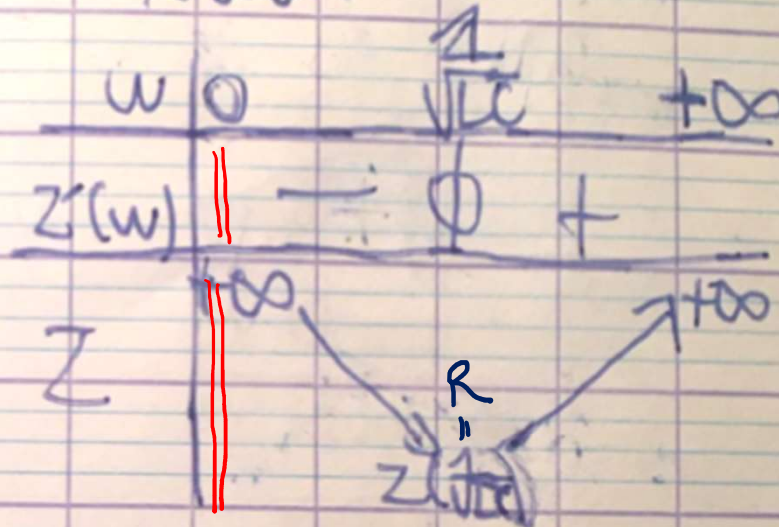
$$\Leftrightarrow \omega^2 \geq \frac{1}{LC}$$

$$\Leftrightarrow \sqrt{\omega^2} \geq \sqrt{\frac{1}{LC}}$$

$$\Leftrightarrow |\omega| \geq \frac{1}{\sqrt{LC}}$$

$$\Leftrightarrow \omega \geq \frac{1}{\sqrt{LC}}$$

Tallem



$$Z\left(\frac{1}{\sqrt{LC}}\right) = \sqrt{R^2 + \left(\frac{L}{\sqrt{LC}} - \frac{1}{C \frac{1}{\sqrt{LC}}}\right)^2} = R$$

$$\frac{L}{\sqrt{LC}} = \frac{\sqrt{L} \sqrt{L}}{\sqrt{L} \sqrt{C}} = \sqrt{\frac{L}{C}}$$

$$\frac{1}{C \frac{1}{\sqrt{LC}}} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}}$$

$$= \frac{\sqrt{L} \sqrt{L}}{\sqrt{C} \sqrt{C}} = \frac{L}{C}$$

② $I(\omega) = \frac{U}{Z(\omega)} = U \times \frac{1}{Z(\omega)}$ où U est une Cte > 0

$I'(\omega) = \left(\frac{1}{Z}\right)' = \frac{-U \cdot Z'(\omega)}{Z(\omega)^2}$ est du signe opposé de $Z'(\omega)$

Tableau de Z :

ω	0	$\frac{1}{\sqrt{LC}}$	$+\infty$
$Z'(\omega)$	-	0	+
Z	$+\infty$	R	$+\infty$

Note: A vertical red line is drawn at $\omega = 0$. An arrow points from the value R in the Z row to the value $\frac{U}{R}$ in the I' row of the adjacent table.

Tableau de I : $I(\omega) = \frac{U}{Z(\omega)}$

ω	0	$\frac{1}{\sqrt{LC}}$	$+\infty$
$I'(\omega)$		+	0
		$\frac{U}{R}$	-

Note: A vertical triple line is drawn at $\omega = 0$. An arrow points from the value $\frac{U}{R}$ in the I' row to the value 0 in the Z row of the adjacent table.

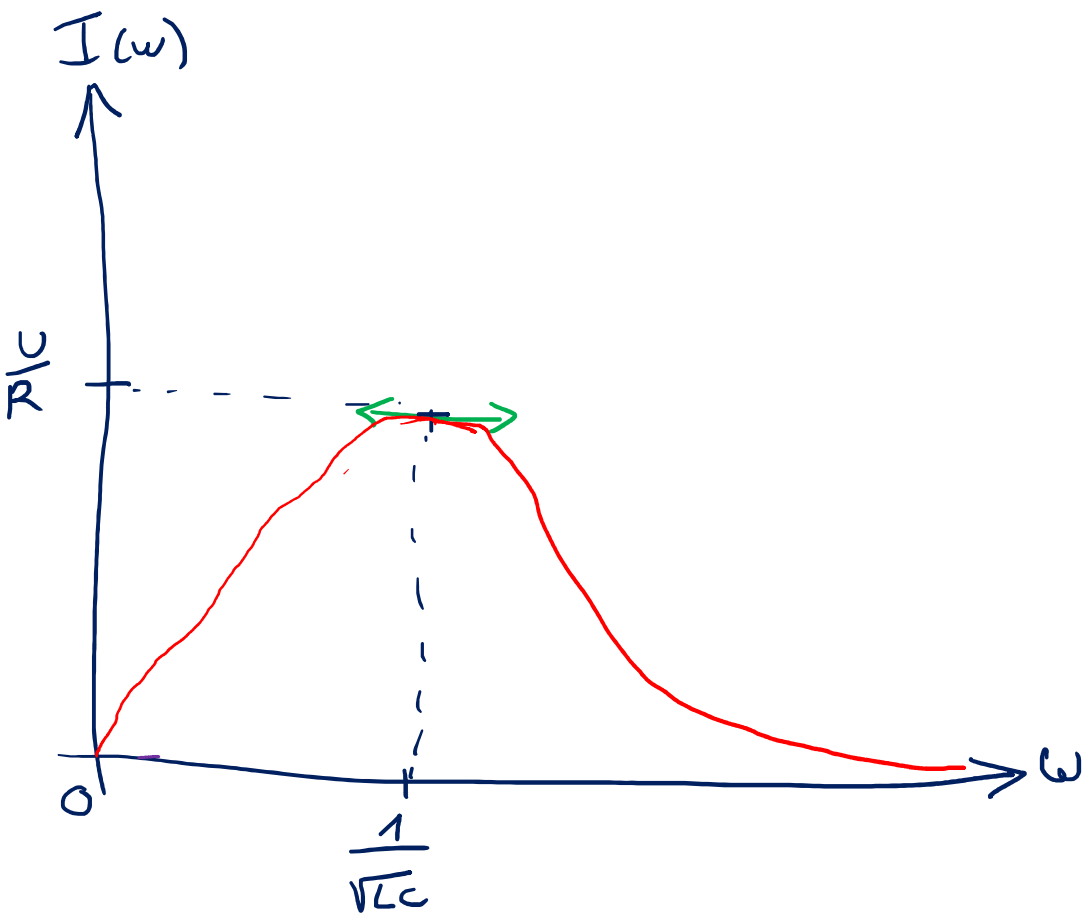


Tableau de I : $I(\omega) = \frac{U}{Z(\omega)}$

ω	0	$\frac{1}{\sqrt{LC}}$	$+\infty$
$I'(\omega)$	$ $	$+$	0
	$ $	0	$-$

Annotations:
 - $\frac{1}{\sqrt{LC}}$ is labeled "tangente horizontale" (horizontal tangent).
 - $+\infty$ is labeled "asymptote".
 - A purple oval encircles the 0 and $-$ signs in the $I'(\omega)$ row.
 - An arrow points from the 0 in the $I'(\omega)$ row to the R/LC label on the graph.
 - Another arrow points from the 0 in the $I'(\omega)$ row to the 0 on the graph's x-axis.

Exercice 4 - chap3

$$R > 0 ; r > 0$$

$$\text{ex 4} \quad P(R) = E^2 \cdot \frac{R}{(R+r)^2}$$

$$P'(R) = \left(\frac{U}{V}\right)' = \frac{U \cdot V' - U' \cdot V}{V^2}$$

$$= E^2 \cdot \frac{(R+r)^2 - R \cdot 2(R+r)}{(R+r)^4}$$

$$\Leftrightarrow p'(R) = E^2 \frac{(R+r)(R+r-2R)}{(R+r)^4}$$

$$p'(R) = E^2 \frac{r-R}{(R+r)^3}$$

$$p'(R) \geq 0 \Leftrightarrow r - R \geq 0$$

$$\Leftrightarrow R \leq r$$

$$P'(R) \geq 0 \Leftrightarrow (R+r)^2 - 2R \cdot (R+r) \geq 0$$

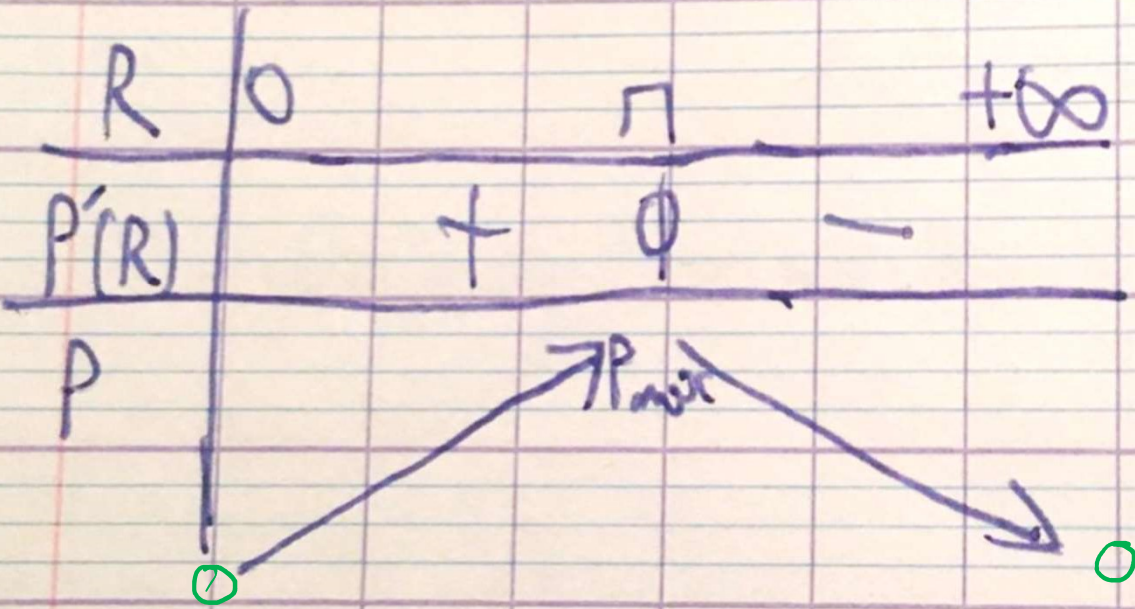
$$\Leftrightarrow R^2 + 2Rr + r^2 - 2R^2 - 2Rr \geq 0$$

$$\Leftrightarrow -R^2 + r^2 \geq 0$$

$$\Leftrightarrow -R^2 + \pi^2 > 0$$

$$\Leftrightarrow \pi^2 > R^2 \Leftrightarrow \pi > R$$

$$p(R) = \epsilon^2 \frac{R}{(R+r)^2}$$



$$P_{\max}(R=\pi) = \epsilon^2 \cdot \frac{\pi}{(\pi+\pi)^2} = \epsilon^2 \cdot \frac{\pi}{(2\pi)^2} = \epsilon^2 \cdot \frac{\pi}{4\pi^2} = \epsilon \cdot \frac{1}{4\pi}$$

$$P_{\max} = \frac{\epsilon^2}{4\pi}$$

est la puissance maximale.