

Travaux Pratiques sur les séries de Fourier

1) Prise en main de Mathcad

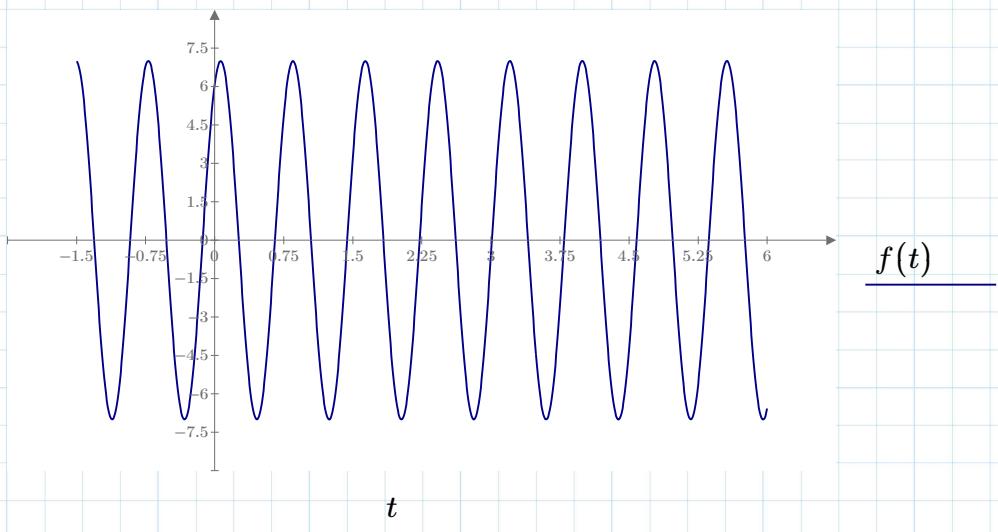
$$\frac{16 - 5 \cdot 2}{\sqrt{3} + 4} = 1.884$$

$$\frac{16 - 5 \cdot 2}{\sqrt{3} + 4} \rightarrow \frac{10.8}{\sqrt{3} + 4}$$

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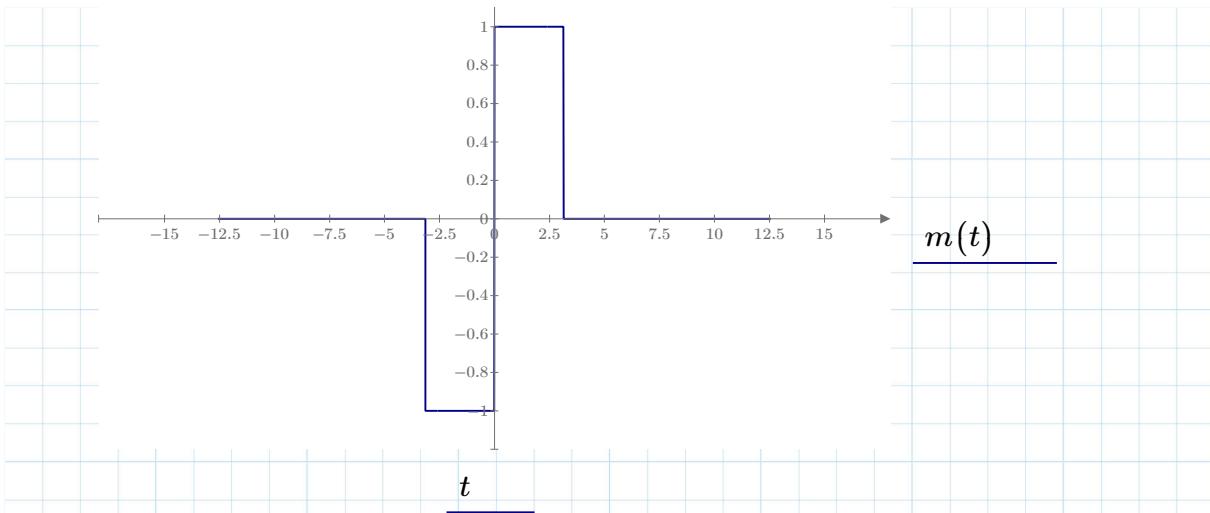
$$f(t) := 7 \cdot \sin\left(8 \cdot t + \frac{\pi}{3}\right)$$

$$t := -1.5, -1.5 + 0.01..6$$

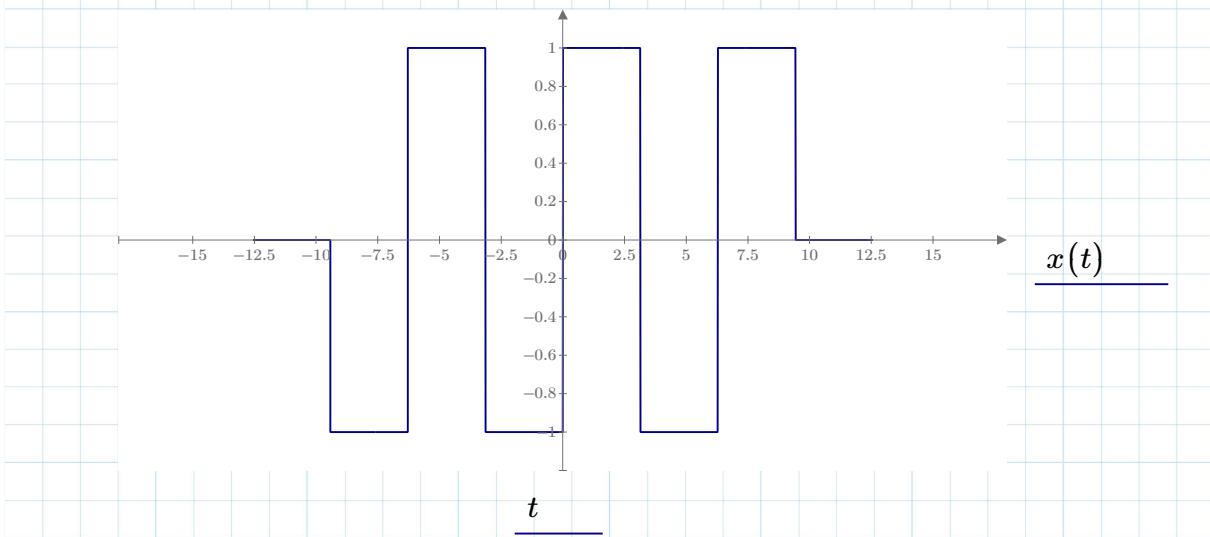


2) Résolution de l'exercice page 7 du chapitre 8

$$m(t) := \begin{cases} \text{if } t < -\pi \\ \quad \| 0 \\ \text{if } -\pi \leq t < 0 \\ \quad \| -1 \\ \text{if } 0 \leq t < \pi \\ \quad \| 1 \\ \text{if } t \geq \pi \\ \quad \| 0 \end{cases} \quad t := -4 \cdot \pi, -4 \cdot \pi + 0.01..4 \cdot \pi$$



$$x(t) := m(t + 2 \cdot \pi) + m(t) + m(t - 2 \cdot \pi)$$



Coefficients de Fourier de x

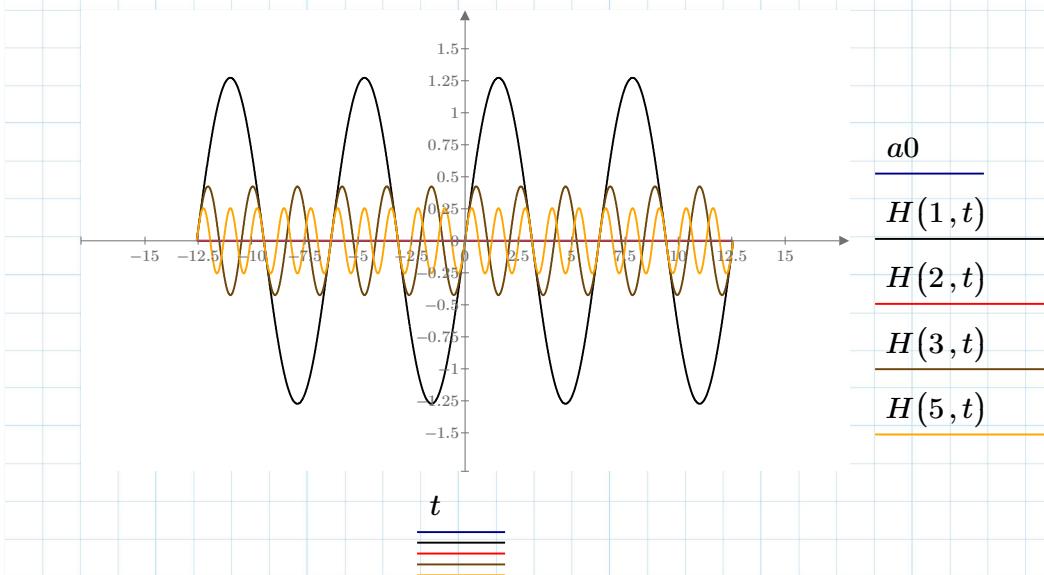
$$T := 2 \cdot \pi \quad \omega := 2 \cdot \frac{\pi}{T} \quad t_0 := 0$$

$$a_0 := \frac{1}{T} \cdot \int_{t_0}^{t_0+T} x(t) dt \quad a(n) := \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \cos(n \cdot \omega \cdot t) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \sin(n \cdot \omega \cdot t) dt$$

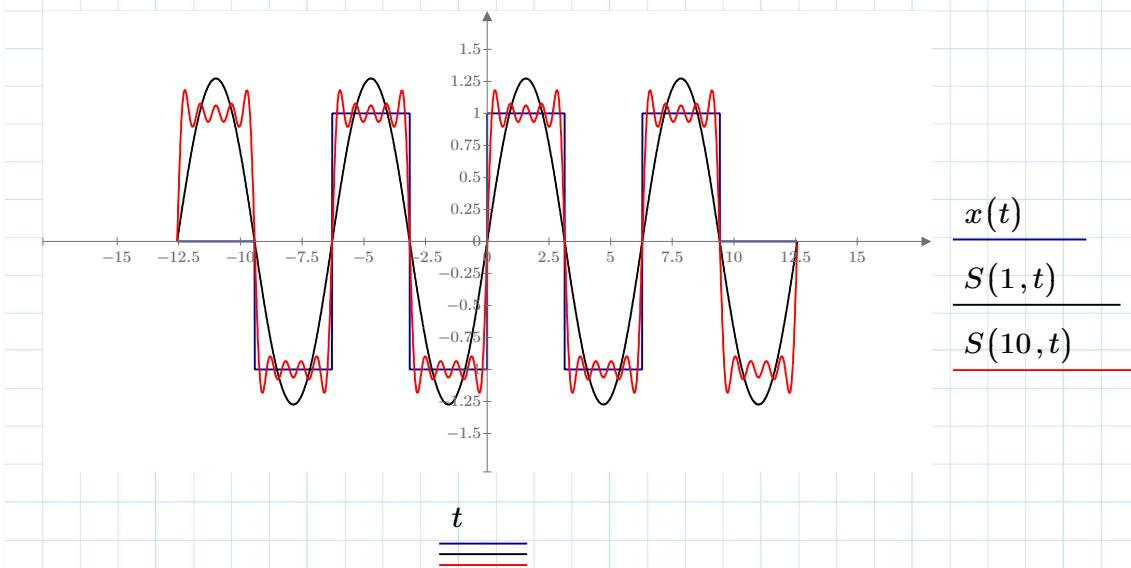
Valeur moyenne, fondamental, harmonique

$$H(n, t) := a(n) \cdot \cos(n \cdot \omega \cdot t) + b(n) \cdot \sin(n \cdot \omega \cdot t)$$



Série de Fourier de rang N

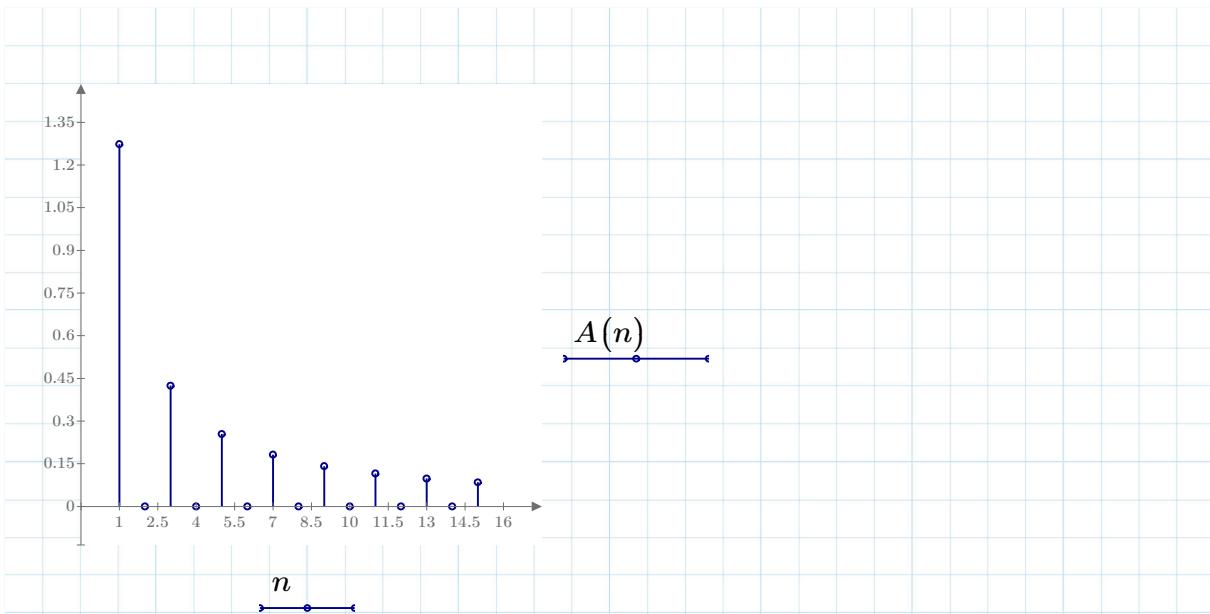
$$S(N, t) := a_0 + \sum_{n=1}^N H(n, t)$$



Spectre d'amplitude de x

$$A(n) := \sqrt{a(n)^2 + b(n)^2}$$

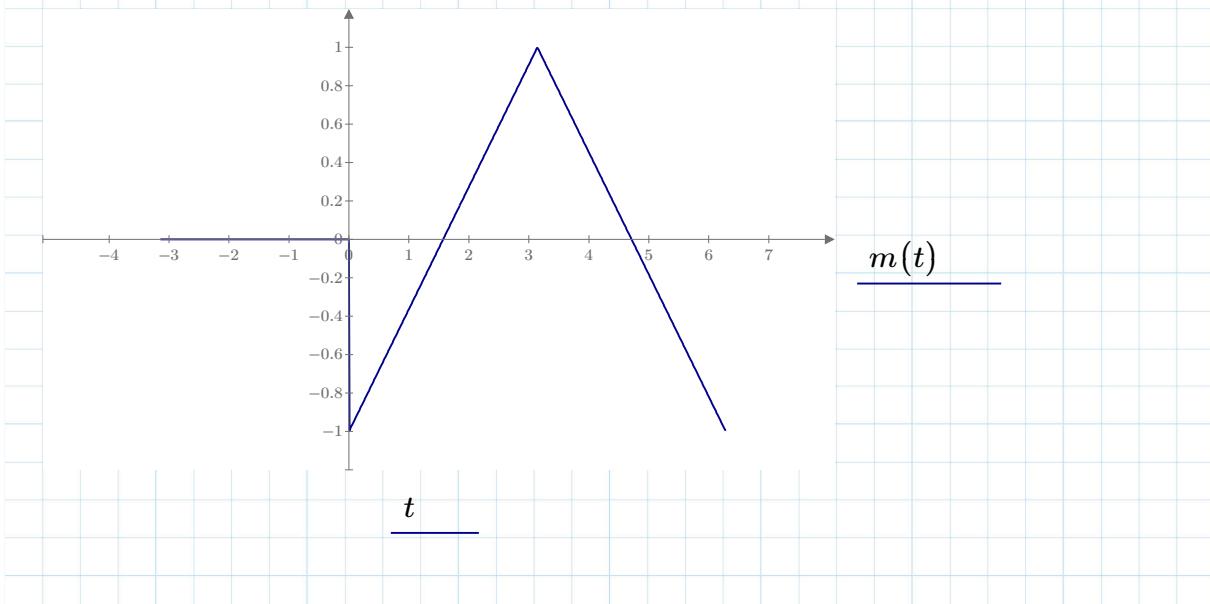
$$n := 1, 2 \dots 15$$



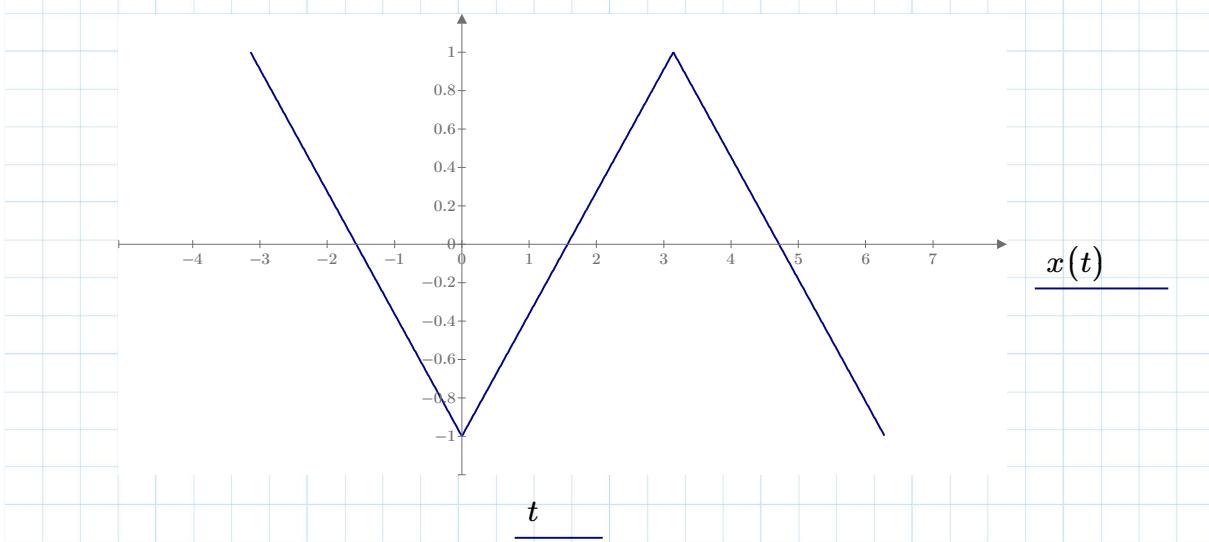
3) Autres séries de Fourier : page 14 a) Signal triangle

$$m(t) := \begin{cases} 0 & \text{if } t < 0 \\ \frac{-2}{\pi} \cdot t + 3 & \text{if } \pi \leq t < 2 \cdot \pi \\ \frac{2}{\pi} \cdot t - 1 & \text{if } 0 \leq t < \pi \\ 0 & \text{if } t \geq 2 \cdot \pi \end{cases}$$

$t := -\pi, -\pi + 0.01 \dots 2 \cdot \pi$



$$x(t) := m(t+2\cdot\pi) + m(t) + m(t-2\cdot\pi)$$



Coefficients de Fourier de x

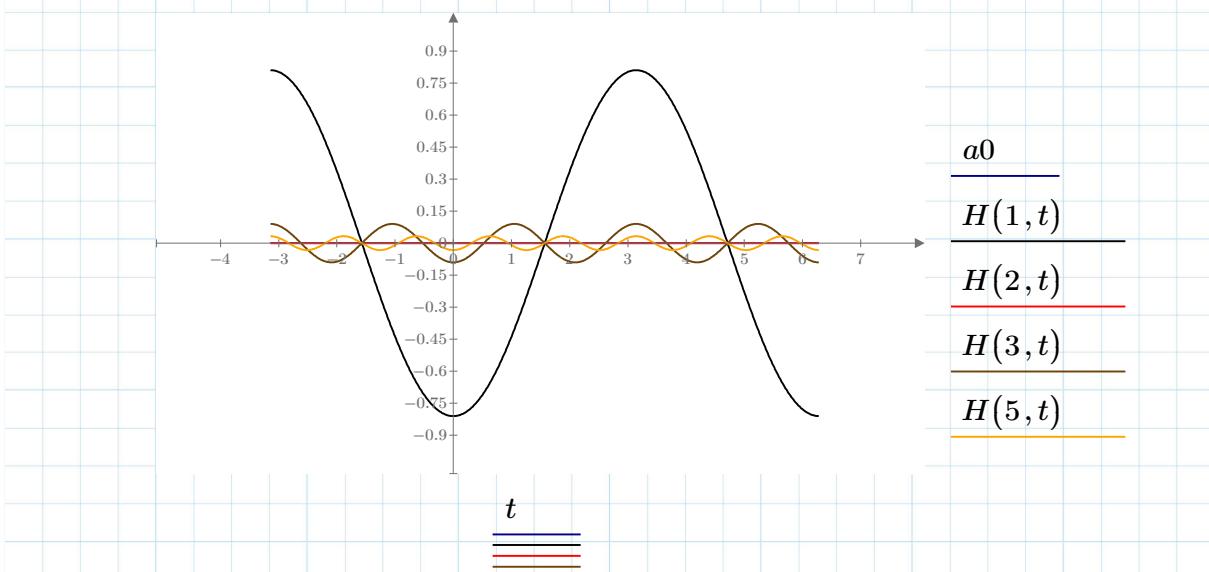
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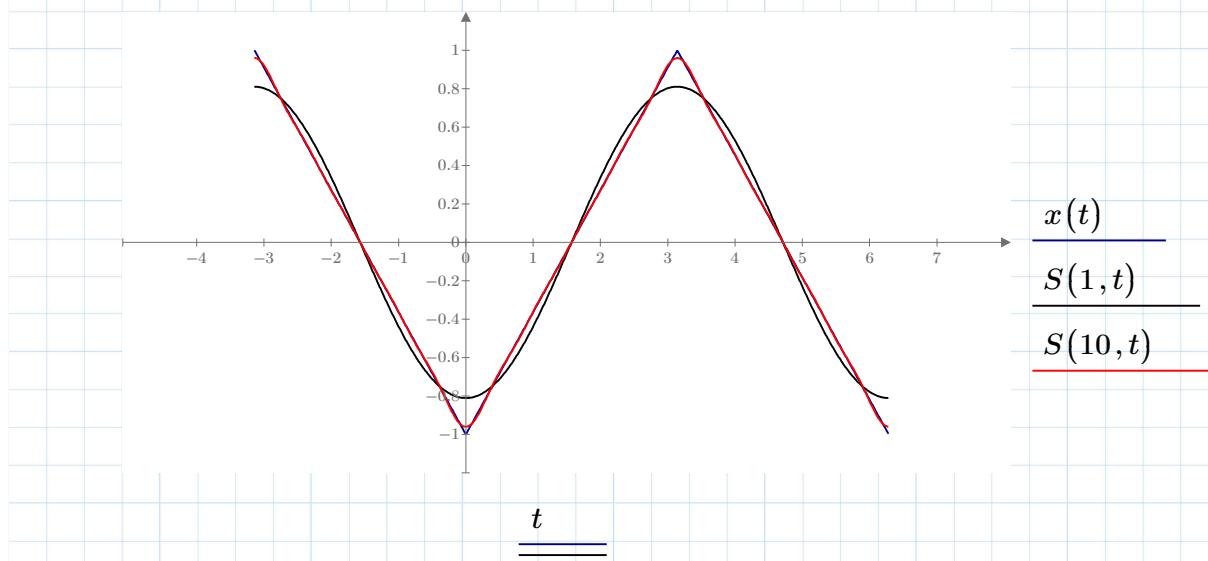
Valeur moyenne, fondamental, harmonique

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Série de Fourier de rang N

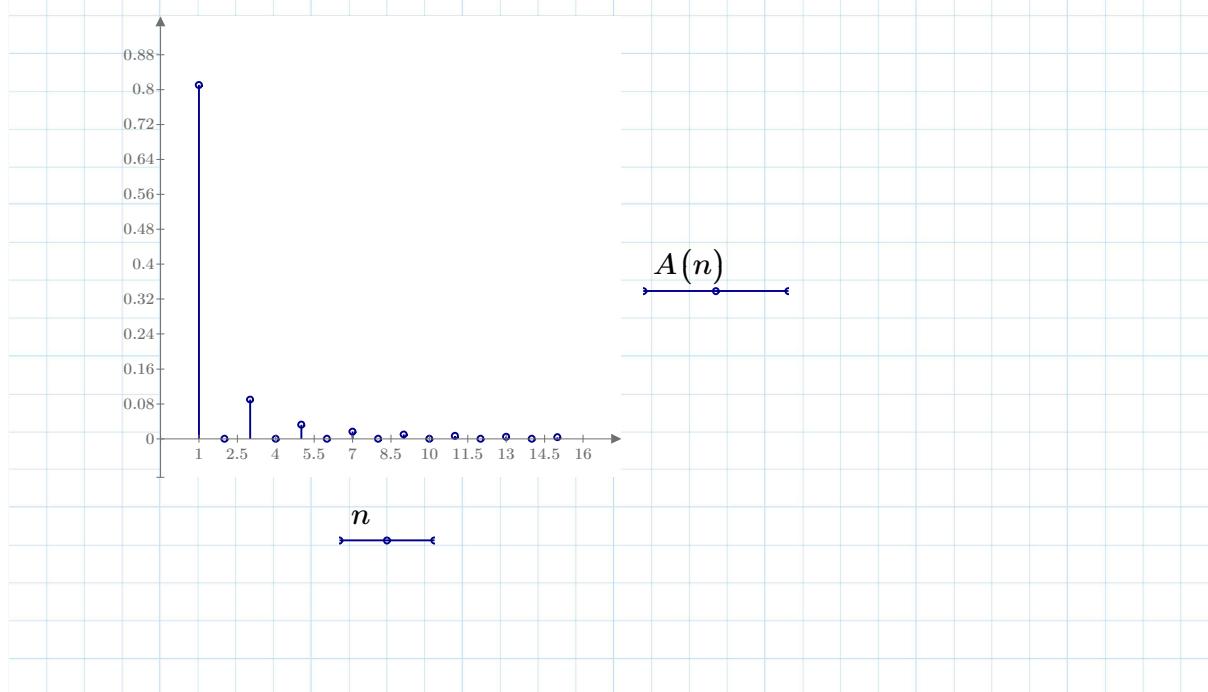
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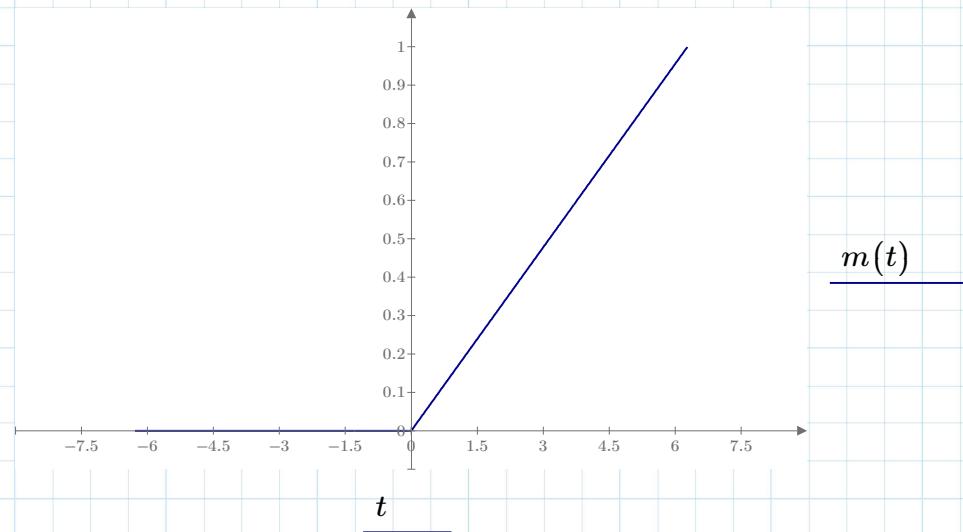
$n := 1, 2..15$



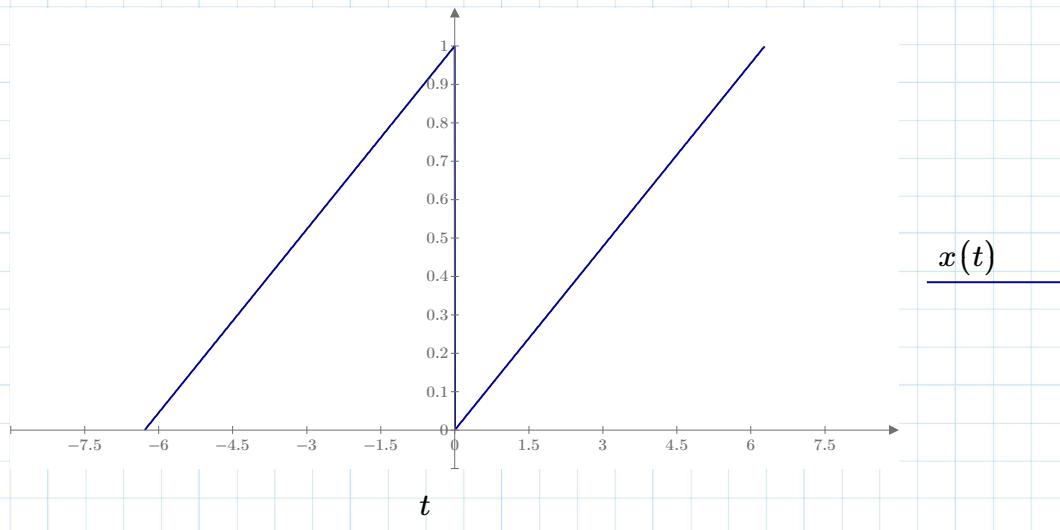
b) Signal en dents de scie

$$m(t) := \begin{cases} \text{if } 0 \leq t < 2 \cdot \pi \\ \frac{1}{2 \cdot \pi} \cdot t \\ \text{else} \\ 0 \end{cases}$$

$t := -2 \cdot \pi, -2 \cdot \pi + 0.01 \dots 2 \cdot \pi$



$$x(t) := m(t + 2 \cdot \pi) + m(t) + m(t - 2 \cdot \pi)$$



Coefficients de Fourier de x

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$$\omega := 2 \cdot \frac{\pi}{T}$$

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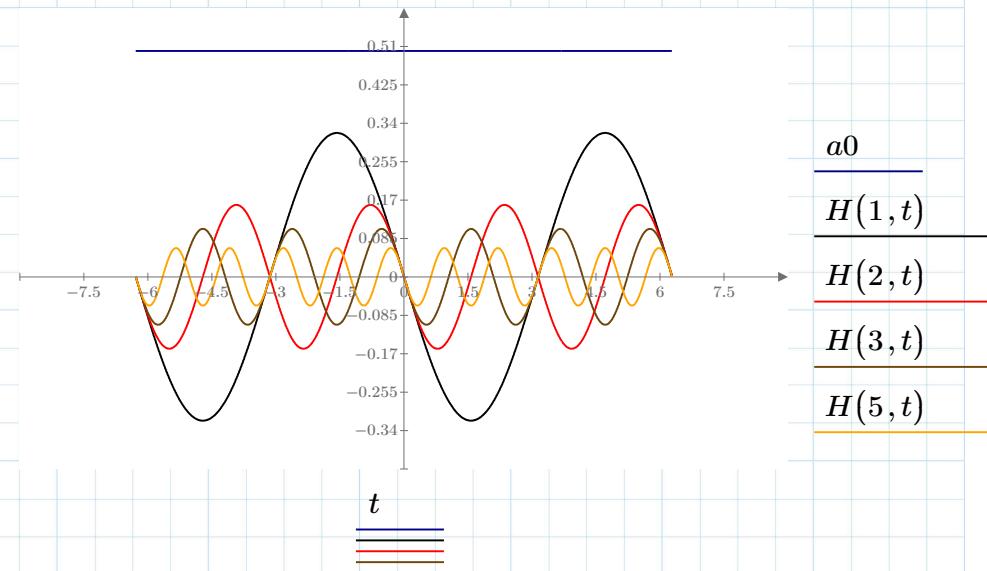
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$$b(n) := \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \sin(n \cdot \omega \cdot t) dt$$

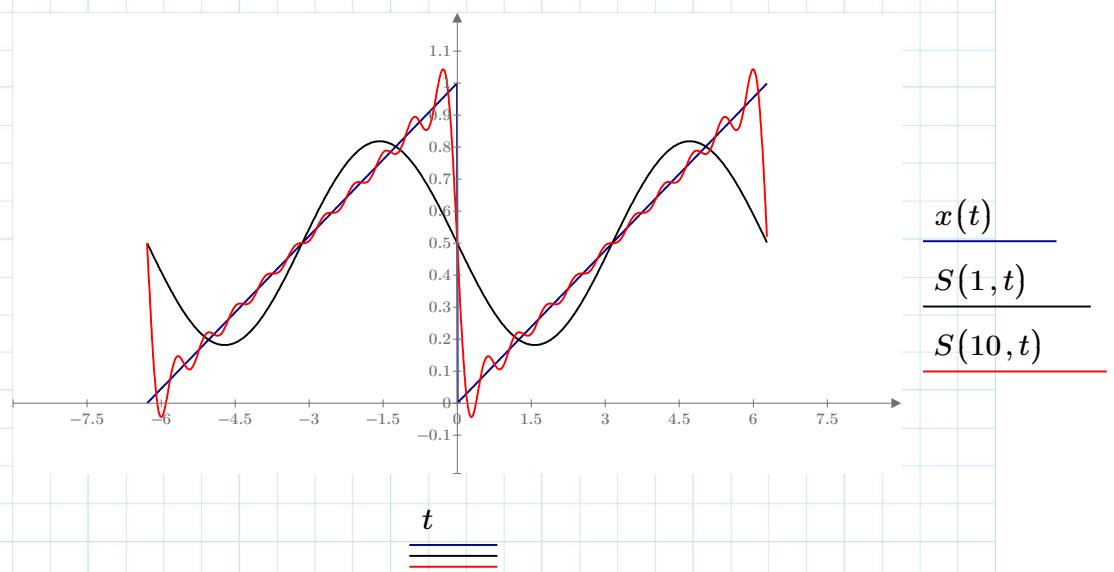
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