

TP2

Ex2

$$3s'(t) + 2s(t) = 2e(t)$$

$$\textcircled{1} 6j\pi f S(f) + 2S(f) = 2E(f)$$

$$S(f) \cdot (6j\pi f + 2) = 2E(f)$$

$$H(f) = \frac{S(f)}{E(f)} = \frac{2}{6j\pi f + 2}$$

$$H(f) = \frac{1}{3j\pi f + 1}$$

$$\textcircled{2} h(t) = \mathcal{T}_F^{-1}(H(f))$$

$$H(f) = \frac{1}{3j\pi f + 1} \times \frac{\frac{2}{3}}{\frac{2}{3}}$$

$$H(f) = \frac{2}{3} \frac{1}{\frac{2}{3} + 2j\pi f}$$

$$h(t) = \frac{2}{3} \cdot e^{-\frac{2}{3}t} \cdot U(t)$$

③ Réponse impulsionnelle :
lorsque $e(t) = \delta(t)$ alors

$$E(f) = 1$$

$$S(f) = H(f) \times E(f)$$

$$S(f) = H(f)$$

$$s(t) = h(t) = \frac{2}{3} e^{-\frac{2}{3}t} U(t) \text{ est}$$

donc la réponse impulsionnelle

④ $e(t) = \text{rect}(t)$

$$S(f) = H(f) \times E(f)$$

$$s(t) = h(t) * e(t)$$

↑
produit de convolution

$$s(t) = \frac{2}{3} e^{-\frac{2}{3}t} U(t) * \text{rect}(t)$$

$$s(t) = \frac{2}{3} \int_{-\infty}^{+\infty} \text{rect}(u) e^{-\frac{2}{3}(t-u)} U(t-u) du$$

$$\lambda(t) = \frac{2}{3} \int_{-1/2}^{t-1/2} e^{-\frac{2}{3}(t-u)} \cdot U(t-u) du$$

On pose $\vartheta = t - u$

u varie entre $-1/2 \Leftrightarrow \vartheta = t + 1/2$

et $1/2 \Leftrightarrow \vartheta = t - 1/2$

$$d\vartheta = -du$$

$$\lambda(t) = \frac{2}{3} \int_{t+1/2}^{t-1/2} e^{-\frac{2}{3}\vartheta} U(\vartheta) d\vartheta$$

$$\lambda(t) = \frac{2}{3} \int_{t-1/2}^{t+1/2} e^{-\frac{2}{3}\vartheta} U(\vartheta) d\vartheta$$

1^{er} Cas $t + 1/2 < 0 \Leftrightarrow t < -1/2$

$$\lambda(t) = 0$$

2^e Cas $t - 1/2 < 0 < t + 1/2$

$$\Leftrightarrow -\frac{1}{2} < t < \frac{1}{2}$$

$$\lambda(t) = \frac{2}{3} \int_0^{t+1/2} e^{-\frac{2}{3}v} dv$$

$$\lambda(t) = \left[-e^{-\frac{2}{3}v} \right]_0^{t+1/2}$$

$$\lambda(t) = 1 - e^{-\frac{2}{3}(t+1/2)}$$

3^{es} $t - 1/2 > 0 \Leftrightarrow t > 1/2$

$$\lambda(t) = \frac{2}{3} \int_{t-1/2}^{t+1/2} e^{-\frac{2}{3}v} dv$$

$$\lambda(t) = \left[-e^{-\frac{2}{3}v} \right]_{t-1/2}^{t+1/2}$$

$$\lambda(t) = e^{-\frac{2}{3}(t-1/2)} - e^{-\frac{2}{3}(t+1/2)}$$

$$= e^{-\frac{2}{3}t} \left(e^{1/3} - e^{-1/3} \right)$$

$$\lambda(t) = 2e^{-\frac{2}{3}t} \operatorname{sh}\left(\frac{1}{3}\right)$$

la réponse est donc :

$$s(t) = \begin{cases} 0 & \text{si } t \leq -1/2 \\ 1 - e^{-\frac{2}{3}(t+\frac{1}{2})} & \text{si } -\frac{1}{2} < t < \frac{1}{2} \\ 2 e^{-\frac{2}{3}t} \phi_n(1/3) & t \geq \frac{1}{2} \end{cases}$$

Ex 4

$$(f * g)(t) = \int_{-\infty}^{\infty} e^{3u} \text{rect}(u) e^{-4(t-u)} \cdot U(t-u) du$$

$$(f * g)(t) = \int_{-1/2}^{1/2} e^{3u} e^{-4(t-u)} \cdot U(t-u) du$$

On pose $\theta = t - u$ $d\theta = -du$

$u = -1/2 \Rightarrow \theta = t + 1/2$

$u = 1/2 \Rightarrow \theta = t - 1/2$

$$(f * g)(t) = \int_{t+1/2}^{t-1/2} e^{3(t-\theta)} e^{-4\theta} \cdot U(\theta) d\theta$$

$$(f * g)(t) = \int_{t-1/2}^{t+1/2} e^{3t} \cdot e^{-7v} \cdot v(v) dv$$

$$(f * g)(t) = e^{3t} \int_{t-1/2}^{t+1/2} e^{-7v} \cdot v(v) dv$$

1^o Cas $t+1/2 < 0 \Leftrightarrow t < -1/2$

$$(f * g)(t) = 0$$

2^o Cas $t-1/2 < 0 < t+1/2 \Leftrightarrow -1/2 < t < 1/2$

$$(f * g)(t) = e^{3t} \int_0^{t+1/2} e^{-7v} dv$$

$$= e^{3t} \left[\frac{e^{-7v}}{-7} \right]_0^{t+1/2}$$

$$(f * g)(t) = \frac{e^{3t}}{7} \left(1 - e^{-7(t+1/2)} \right)$$

3^o Cas $t-1/2 > 0 \Leftrightarrow t > 1/2$

$$(f * g)(t) = e^{3t} \int_{t-1/2}^{t+1/2} e^{-7v} dv$$

$$(f+g)(t) = e^{3t} \cdot \left[\frac{e^{-7t}}{-7} \right]^{t+1/2}$$

$$(f+g)(t) = \frac{e^{3t}}{7} \cdot \left(e^{-7(t-1/2)} - e^{-7(t+1/2)} \right)$$

$$(f+g)(t) = \frac{e^{-4t}}{7} \left(e^{7/2} - e^{-7/2} \right)$$

$$(f+g)(t) = \frac{2e^{-4t}}{7} \sinh(7/2)$$