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GEII

TP 7 Maths Outils et Logiciels
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Exercice 1 :

1) La méthode longue :

a) $\dfrac{a}{x+1} + \dfrac{b}{x-2} = \dfrac{a(x-2)+b(x+1)}{(x+1)(x-2)} = \dfrac{ax-2a+bx+b}{(x+1)(x-2)}$

b) $7-2x = ax-2a+bx+b$

$\Leftrightarrow 7-2x = x(a+b)-2a+b$

$\Leftrightarrow \begin{cases} -2 = a+b \\ 7 = -2a+b \end{cases} \Leftrightarrow \begin{cases} a = -2-b \\ 7 = -2(-2-b)+b \end{cases} \Leftrightarrow \begin{cases} a = -2-b \\ 7 = 4+3b \end{cases}$

$\Leftrightarrow \begin{cases} a = -2-b \\ b = 1 \end{cases} \Leftrightarrow \begin{cases} a = -3 \\ b = 1 \end{cases}$

Conclusion : Donc $f(x) = \dfrac{-3}{(x+1)} + \dfrac{1}{(x-2)}$

2) La méthode courte :

a) $\lim\limits_{x\to -1} (x+1) f(x) = \left[(x+1)\times f(x)\right]_{x=-1}$

$a = \left[\dfrac{\cancel{(x+1)}\times(7-2x)}{\cancel{(x+1)}\times(x-2)}\right]_{x=-1} = \left[\dfrac{7-2x}{(x-2)}\right]_{x=-1} = \left[\dfrac{7-2\times -1}{(-1-2)}\right] = \dfrac{9}{-3}$

Donc $a = -3$

b) $b = \left[\dfrac{(x-2)(7-2x)}{(x-2)(x+1)}\right]_{x=2} = \left[\dfrac{7-2x}{x+1}\right]_{x=2} = \left[\dfrac{7-2\times 2}{2+1}\right] = \dfrac{3}{3}$

Donc $b = 1$

c) Conclusion : $f(x) = \dfrac{-3}{(x+1)} + \dfrac{1}{(x-2)}$

$(\ln(|u|))' = \dfrac{u'}{u}$

d) $F(x) = -3\ln(|x+1|) + \ln(|x-2|) + Cte$

Exercice 2 :

a) $g(x) = \dfrac{2x+3}{(x^2-x-6)(x-2)} = \dfrac{2x+3}{(x+2)(x-3)(x-2)}$

$x^2 - x - 6 = 0$

$\Delta = b^2 - 4ac$

$\Delta = 1^2 - 4 \times 1 \times (-6)$

$\Delta = 1 - 4 \times -6$

$\Delta = 1 + 24 = 25$

$\Delta > 0$ donc 2 racines conjuguées

$x1 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{1+5}{2} = \dfrac{6}{2} = 3$

$x2 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{1-5}{2} = \dfrac{-4}{2} = -2$

$D_g = \mathbb{R} \setminus \{-2 ; 2 ; 3\}$

g(x) est irréductible car il n'y pas de facteur commun entre le numérateur et le dénominateur.

b) $g(x) = \dfrac{a}{x+2} + \dfrac{b}{x-3} + \dfrac{c}{x-2}$

c) <u>On utilise la méthode courte</u> : $a = \lim\limits_{x \to -2} (x+2) \times g(x)$

$b = \lim\limits_{x \to 3} (x-3) \times g(x)$

$c = \lim\limits_{x \to 2} (x-2) \times g(x)$

$a = \left[\dfrac{\cancel{(x+2)} \times (2x+3)}{\cancel{(x+2)}(x-3)(x-2)}\right]_{x=-2} = \left[\dfrac{2x+3}{(x-3)(x-2)}\right]_{x=-2} = \left[\dfrac{-4+3}{-5 \times (-4)}\right]_{x=-}$

$\boxed{a = \dfrac{-1}{20}}$

$b = \left[\dfrac{\cancel{(x-3)}(2x+3)}{\cancel{(x-3)}(x+2)(x-2)}\right]_{x=3} = \left[\dfrac{2x+3}{(x+2)(x-2)}\right]_{x=3} = \left[\dfrac{6+3}{5 \times 1}\right]$

$\boxed{b = \dfrac{9}{5}}$

$c = \left[\dfrac{\cancel{(x-2)}(2x+3)}{\cancel{(x-2)}(x+2)(x-2)}\right]_{x=2} = \left[\dfrac{2x+3}{(x-3)(x+2)}\right]_{x=2} = \left[\dfrac{4+3}{-1 \times 4}\right]$

$\boxed{c = -\frac{7}{4}}$

<u>Conclusion :</u> $g(x) = \frac{-1}{20(x+2)} + \frac{9}{5(x-3)} - \frac{7}{4(x-2)}$

d) $G(x) = \frac{1}{20}\ln(|x+2|) + \frac{9}{5}\ln(|x-3|) - \frac{7}{4}\ln(|x-2|) + Cte$

<u>Exercice 3 :</u>

1) $r(t) = \frac{t-1}{(t^2-1)(t+2)} = \frac{\cancel{t-1}}{(t+1)\cancel{(t-1)}(t+2)} = \frac{1}{(t+1)(t+2)}$

$a^2 - b^2 = (a+b)(a-b)$

$r(t) = \frac{a}{t+1} + \frac{b}{t+2}$

$\mathcal{D}r = \mathbb{R} \setminus \{-2 ; -1\}$

$a = \lim_{t \to -1} (t+1) \times r(t)$
$b = \lim_{t \to -2} (t+2) \times r(t)$

$a = \left[\frac{(t+1)}{(t+1)(t+2)}\right]_{t \to -1} = \left[\frac{1}{t+2}\right]_{t \to -1} = \left[\frac{1}{t+2}\right]_{x \to -1}$

$a = \left[\frac{1}{2-1}\right]_{t \to -1} = \; = 1 \qquad \boxed{a = 1}$

$b = \left[\frac{(t+2)}{(t+2)(t+1)}\right] = \left[\frac{1}{-2+1}\right] = -1 \qquad \boxed{b = -1}$

<u>Conclusion :</u> $r(t) = \frac{1}{t+1} - \frac{1}{t+2}$

$R(t) = \ln(|t+1|) - \ln(|t+2|) + Cte$

2) $\int_{-4}^{-3} r(t) = \Big[\ln(|t+1|) - \ln(|t+2|)\Big]_{-4}^{-3}$

$\int_{-4}^{-3} r(t) = \Big(\ln(|-3+1|) - \ln(|-3+2|)\Big) - \Big(\ln(|-4+1|) - \ln(|-4+2|)\Big)$

$$\int_{-4}^{-3} r(t) = \left(\ln(2) - \ln(1)\right) - \left(\ln(3) - \ln(2)\right)$$

$$\int_{-4}^{-3} r(t) = \ln(2) - \underbrace{\ln(1)}_{0} - \ln(3) + \ln(2)$$

$$\boxed{\int_{-4}^{-3} r(t) = 2\ln(2) - \ln(3)}$$