

EXOS SUP
LAPLACE

TP Tableaux blcs

$$\begin{cases} \lambda'(t) - \lambda(t) = 5e'(t) + e(t) \\ e(0) = \lambda(0) = 0 \end{cases}$$

1) Fonction de transfert : $H(p)$

2) Réponse impulsionnelle

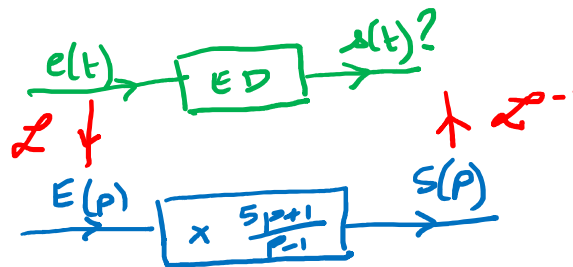
3) " indicielle

4) Si $e(t) = 3\cos(t)$, alors $\lambda(t) = ?$

1) $pS(p) - S(p) = 5pE(p) + E(p)$

$$(p-1) \cdot S(p) = (5p+1) \cdot E(p)$$

$$H(p) = \frac{S(p)}{E(p)} = \frac{5p+1}{p-1}$$



Notes

2) Si $e(t) = \delta(t)$ Alors $E(p) = 1$ et $S(p) = H(p) \cdot E(p)$

$S(p) = \frac{5p+1}{p-1}$ a une partie entière

$$\begin{array}{l|l} A = 5p+1 & p-1 = B \\ - (5p-5) & 5 = Q \\ \hline R = 6 & \end{array}$$

donc $S(p) = 5 + \frac{6}{p-1}$

$$\frac{A}{B} = Q + \frac{R}{B}$$

et $\lambda(t) = 5\delta(t) + 6e^t$ est la réponse impulsionnelle.

3) Si $e(t) = 1$ Alors $E(p) = \frac{1}{p}$ et $S(p) = H(p) \cdot E(p) = \frac{5p+1}{p-1} = \frac{a}{p} + \frac{b}{p-1}$

$$a = \left[p \cdot S(p) \right]_{p=0} = \left[\frac{5p+1}{p-1} \right]_{p=0} = -1 \quad \text{et} \quad b = \left[(p-1) \cdot S(p) \right]_{p=1} = \left[\frac{5p+1}{p} \right]_{p=1} = 6$$

Donc $S(p) = \frac{-1}{p} + \frac{6}{p-1}$ et $\lambda(t) = -1 + 6e^t$ est la réponse indicielle.

Notes 4) $e(t) = 3\cos(2t)$ alors $E(p) = 3 \frac{p}{p^2+4} = \frac{3p}{p^2+4}$

$$S(p) = H(p) \cdot E(p) = \frac{3p(5p+1)}{(p-1)(p^2+4)} = \frac{a}{p-1} + \frac{b p + c}{p^2+4}$$

$$a = [(p-1) \cdot S(p)]_{p=1} = \left[\frac{3p(5p+1)}{p^2+4} \right]_{p=1} = \frac{3 \times 6}{5} = \frac{18}{5}$$

$$b \cdot 2j + c = [(p^2+4) \cdot S(p)]_{p=2j} = \left[\frac{3p(5p+1)}{p-1} \right]_{p=2j} = \frac{6j(10j+1)}{2j-1} \times \frac{-2j-1}{-2j-1}$$

$$b \cdot 2j + c = \frac{-60 + 6j}{2^2 + 1^2} = \frac{-60 + 6j}{5} \Leftrightarrow \begin{cases} b = \frac{3}{5} \\ c = -12 \end{cases}$$

Donc:

$$S(p) = \frac{18}{5} \frac{1}{p-1} + \frac{3}{5} \frac{p}{p^2+4} - \frac{12}{2} \frac{1 \times 2}{p^2+4}$$

$$s(t) = \frac{18}{5} e^t + \frac{3}{5} \cos(2t) - 6 \sin(2t) \text{ est la réponse harmonique}$$