EXOS SUP

LAPLACE

TP Tableaux blcs

$$\begin{cases} s'(t) - s(t) = 5e'(t) + e(t) \\ e(0) = s(0) = 0 \end{cases}$$

1) Fonction de transfert : $H(p)$

2) Réponse impulsionnelle

3) " indicielle

4) Si $e(t) = 3\cos(2t)$, alors $s(t) = ?$

1) $pS(p) - S(p) = 5pE(p) + E(p)$

$(p-1).S(p) = (5p+1).E(p)$

$$H(p) = \frac{S(p)}{E(p)} = \frac{5p+1}{p-1}$$

$$\begin{tikzcd}
e(t) \arrow[r] \arrow[d, "\mathcal{L}"'] & \boxed{ED} \arrow[r] & s(t)? \\
E(p) \arrow[r] & \boxed{\times \frac{5p+1}{p-1}} \arrow[r] & S(p) \arrow[u, "\mathcal{L}^{-1}"']
\end{tikzcd}$$

Notes

2) Si $e(t) = \delta(t)$ Alors $E(p) = 1$ et $S(p) = H(p).E(p)$

$S(p) = \dfrac{5p+1}{p-1}$ a une partie entière

$$\begin{array}{l|l} A = 5p+1 & p-1 = B \\ \hline -(5p-5) & 5 = Q \\ \hline R = 6 & \end{array}$$

donc $S(p) = 5 + \dfrac{6}{p-1}$

$\dfrac{A}{B} = Q + \dfrac{R}{B}$

et $s(t) = 5\delta(t) + 6e^{t}$ est la réponse impulsionnelle.

3) Si $e(t) = 1$ Alors $E(p) = \dfrac{1}{p}$ et $S(p) = H(p).E(p) = \dfrac{5p+1}{p(p-1)} = \dfrac{a}{p} + \dfrac{b}{p-1}$

$a = \left[p.S(p)\right]_{p=0} = \left[\dfrac{5p+1}{p-1}\right]_{p=0} = -1$ et $b = \left[(p-1).S(p)\right]_{p=1} = \left[\dfrac{5p+1}{p}\right]_{p=1} = 6$

Donc $S(p) = \dfrac{-1}{p} + \dfrac{6}{p-1}$ et $s(t) = -1 + 6e^{t}$ est la réponse indicielle.

Notes 4) $e(t) = 3\cos(2t)$ alors $E(p) = 3\dfrac{p}{p^2+4} = \dfrac{3p}{p^2+4}$

$$S(p) = H(p).E(p) = \frac{3p(5p+1)}{(p-1)(p^2+4)} = \frac{a}{p-1} + \frac{bp+c}{p^2+4}$$

$$a = \left[(p-1).S(p)\right]_{p=1} = \left[\frac{3p(5p+1)}{p^2+4}\right]_{p=1} = \frac{3\times 6}{5} = \frac{18}{5}$$

$$b.2j+c = \left[(p^2+4).S(p)\right]_{p=2j} = \left[\frac{3p(5p+1)}{p-1}\right]_{p=2j} = \frac{6j(10j+1)}{2j-1} \times \frac{-2j-1}{-2j-1}$$

$$b2j+c = \frac{-60+6j}{2^2+1^2} = \frac{-60+6j}{5} \Longleftrightarrow \begin{cases} b = \frac{3}{5} \\ c = -12 \end{cases}$$

Donc:

$$S(p) = \frac{18}{5}\frac{1}{p-1} + \frac{3}{5}\frac{p}{p^2+4} - \frac{12}{2}\cdot\frac{1\times 2}{p^2+4}$$

$$s(t) = \frac{18}{5}e^t + \frac{3}{5}\cos(2t) - 6\sin(2t)$$ est la réponse harmonique