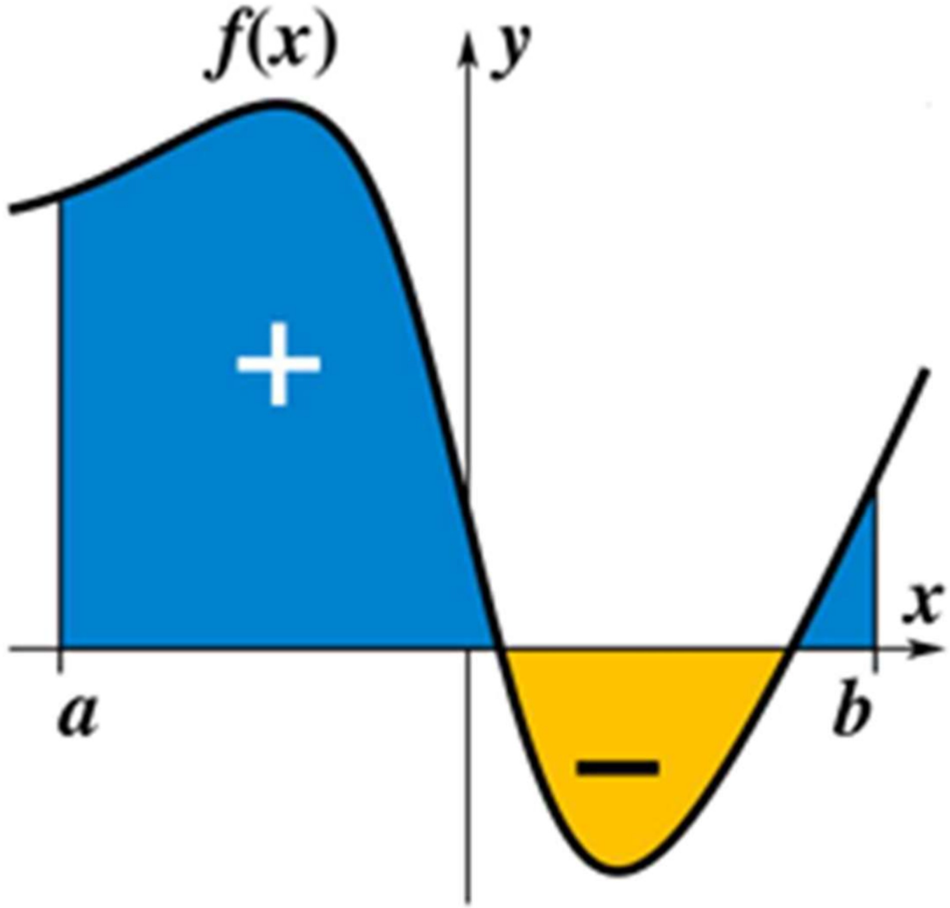
Chapitre 7 : Compléments sur le calcul intégral

Correction DM16



exo 1 p 19

• $P = \int_{-\pi/6}^{\pi/6} \cos(5t)\, dt$

On retrouve une fonction paire car l'intégrale est bornée en 0 $\left[\frac{\pi}{6} ; -\frac{\pi}{6}\right]$ alors on a

$$\int_{-\pi/6}^{\pi/6} \cos(5t)\, dt = 2 \int_{0}^{\frac{\pi}{6}} \cos(5t)\, dt = 2 . \left[\frac{1}{5} \sin 5t\right]_0^{\pi/6} = 2 . \frac{1}{5}\left[\sin \frac{5\pi}{6} - \sin(0)\right]$$

$$= 2 . \frac{1}{10} = \boxed{\frac{1}{5}}$$

$$* \quad K = \int_0^{\pi} \sin(3t) . \cos^8(3t) . dt = \left[-\frac{1}{3} \times \frac{\cos^9(3t)}{9}\right]_0^{\pi}$$

$$K = -\frac{1}{27}\left(\cos^9(3\pi) - \cos^9(0)\right) = -\frac{1}{27}(-1-1)$$

$$K = \frac{2}{27}$$

$$P = \int_0^{\pi/3} \sin\left(2t + \frac{\pi}{3}\right) dt$$

$$\int \sin(at + b)\, dt = -\frac{1}{a} \cos(at + b) + c$$

$a = 2$ et $b = \frac{\pi}{3}$

$$F(t) = -\frac{1}{2} \cos\left(2t + \frac{\pi}{3}\right)$$

$$P = F\left(\frac{\pi}{3}\right) - F(0) = \frac{1}{2} \cos(\pi) + \frac{1}{2} \cos \frac{\pi}{3}$$

$$= \frac{3}{4}$$

Notes

- $H(x) = \int \frac{x+2}{x^2-2x-3}$

$x^2 - 2x - 3 = (x-3)(x+1)$

<u>DSES:</u> $hx = \frac{A}{x-3} + \frac{B}{x+1}$

$A = [(x-3) \times h(x)]_{x=3} = \left[\frac{x+2}{x+1}\right]_{x=3} = \frac{5}{4}$

$B = [(x+1) \times h(x)]_{x=-1} = \left[\frac{(x+2)}{(x-3)}\right]_{x=-1} = -\frac{1}{4}$

- $H(x) = \int \left(\frac{5/4}{x-3} + \frac{-1/4}{x+1}\right) dx = \frac{5}{4} \int \frac{1}{x-3}\, dx - \frac{1}{4} \int \frac{1}{x+1}\, dx = \frac{5}{4} \ln |x-3| - \frac{1}{4} \ln |x+1| + cte$

$\forall\ x \neq 3, x \neq -1$

$\forall\ x \in \mathbb{R} - \{-1; 3\}.$

Notes

$$N(x) = \int \frac{x}{x^3 - x^2 + x - 1}$$

$$x^3 - x^2 + x - 1 = (x^3 + x^2) + (x - 1) = x^2(x+1) + 1(x-1) = (x-1)(x^2+1)$$

$$N(x) = \int \frac{x}{(x-1)(x^2+1)}\,dx$$

On pose $\dfrac{A}{(x-1)} + \dfrac{Bx + C}{x^2+1} = N(x)$

$$A = \left[(x-1) \times N(x)\right]_{x=1} = \left[\frac{x}{x^2+1}\right]_{x=1} = \frac{1}{2}$$

$$x = A(x^2+1) + (Bx + c)(x-1)$$

$$x = \frac{1}{2}(x^2+1) + (Bx + c)(x-1)$$

$$= \frac{1}{2}x^2 + \frac{1}{2} + (Bx + c)(x-1)$$

$$Bj + C = \left[(x^2+1)\,N(x)\right]_{x=j}$$

On pose $x = 0$

$$0 = \frac{1}{2}(0^2+1)(0 + c)(-1)$$

$$c = \frac{1}{2}$$

On pose $x = 2$

$$2 = \frac{1}{2}(2^2+1)(B.2 + c)(2-1)$$

$$b = -\frac{1}{2}$$

On a donc $N(x) = \displaystyle\int \left(\frac{1}{2(x-1)} + \frac{-1/2x + 1/2}{x^2+1} \right) dx$

$$N(x) = \frac{1}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{x}{x^2+1} + \frac{1}{2}\int \frac{1}{x^2+1}\,dx$$

$$N(x) = \frac{1}{2}\ln(x-1) - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\arctan(x) + c$$

$$Q = -\int_{\pi/6}^{\pi/3} -\tan(x)\,dx \qquad \tan x = \frac{\sin x}{\cos x} \longrightarrow \int \frac{u'}{u}\,dx = \ln|u| + cte$$

$$u = \cos x \Rightarrow u' = -\sin x$$

$$Q = \left[-\ln|\cos(x)|\right]_{\pi/6}^{\pi/3} = \underbrace{-\ln\left|\cos\left(\frac{\pi}{3}\right)\right|}_{\ln B} + \ln\left|\cos\frac{\pi}{6}\right| = \ln\left(\frac{\overbrace{\cos\left(\frac{\pi}{6}\right)}^{\frac{\sqrt{3}}{2}}}{\underbrace{\cos\left(\frac{\pi}{3}\right)}_{\frac{1}{2}}}\right)$$

$$\ln A - \ln B = \ln\frac{A}{B}$$

$$Q = \ln\sqrt{3} = \ln\left(3^{1/2}\right) = \frac{1}{2}\ln 3 \qquad \ln x^{\alpha} = \alpha \ln x$$

$$K(x) = \int \underbrace{\left(1+\tan^2(x)\right)}_{u'} . \tan^3(x)\,dx$$

$$\int u' \times u^{\alpha}\,dx = \frac{u^{\alpha+1}}{\alpha+1} + cte$$

$$K(x) = \int \underbrace{\tan(x)^3}_{u^3} . \underbrace{1+\tan^2(x)}_{u'} = \frac{\tan(x)^4}{4} + c$$

$$\frac{u^4}{4}$$

$$\forall x \neq \frac{\pi}{2} + k\pi ;\ k \in \mathbb{Z}$$

- $L(x) = -\int -\frac{1}{x^2} \cdot e^{\frac{1}{x}}\,dx$ $\int u' e^u \,dx = e^u + cte$: $u = 1/x \Rightarrow u' = -1/x^2$

$$\int \frac{1}{x^2} \cdot e^{\frac{1}{x}}\,dx = -\int e^{\frac{1}{x}} \left(-\frac{1}{x^2}\right) dx = -e^{\frac{1}{x}} + C$$

- $Q = \int_0^{\pi/2} \cos(x) \cdot \sin^5(x)\,dx$ $\int u' \cdot u^5 \,dx = \frac{u^6}{6} + cte$.

$u = \sin x \Rightarrow u' = \cos x$

$$Q = \left[\frac{\sin(x)^6}{6}\right]_0^{\pi/2} = \frac{1}{6}$$