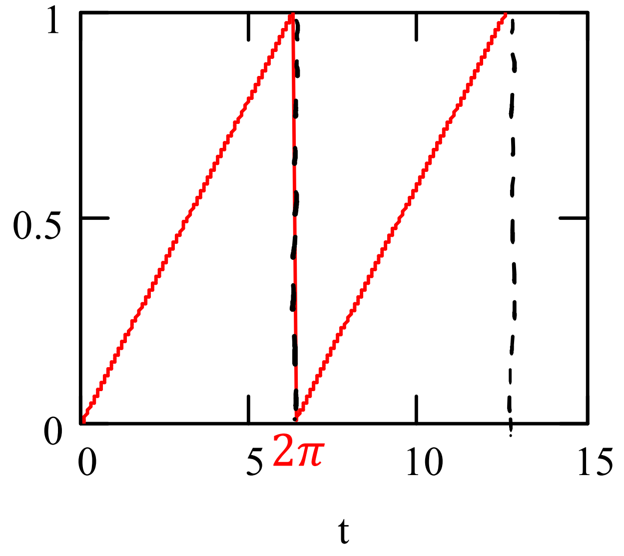
QCM

Calculs de base

Trigonométrie

Nombres complexes

Quelle est l'équation de la droite sur l'intervalle $[0; 2\pi]$? 2'



1. $y = 2\pi t$ — 1%
2. $y = 2\pi / t$ — 2%
3. $y = 2\pi t + 1$ — 3%
4. $y = \frac{1}{2\pi} t$ — 4% ✓4
5. $y = \frac{1}{2\pi} t + 1$ — 5%
6. $y = \frac{1}{2} t$ — 6%

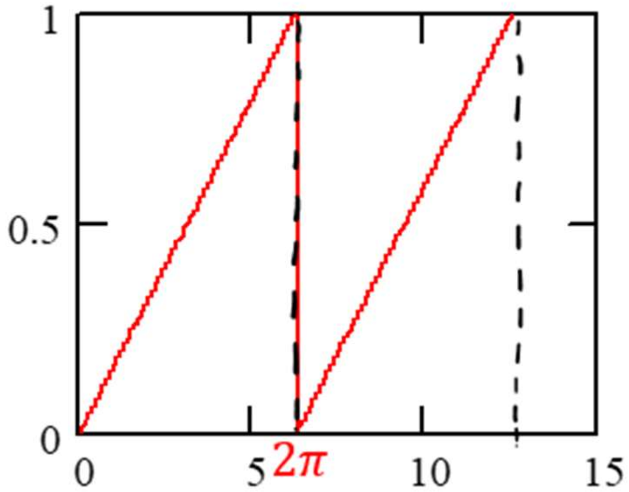
$t \in [0; 2\pi]$

Notes

$\mathcal{D} : y = mt$ car $\mathcal{D}$ passe par O.

$$m = \frac{y_B - y_O}{t_B - t_O} = \frac{1-0}{2\pi - 0} = \frac{1}{2\pi}$$

Donc $\mathcal{D} : y = \frac{1}{2\pi} \cdot t = \frac{t}{2\pi}$



Laquelle de ces formules est juste ?

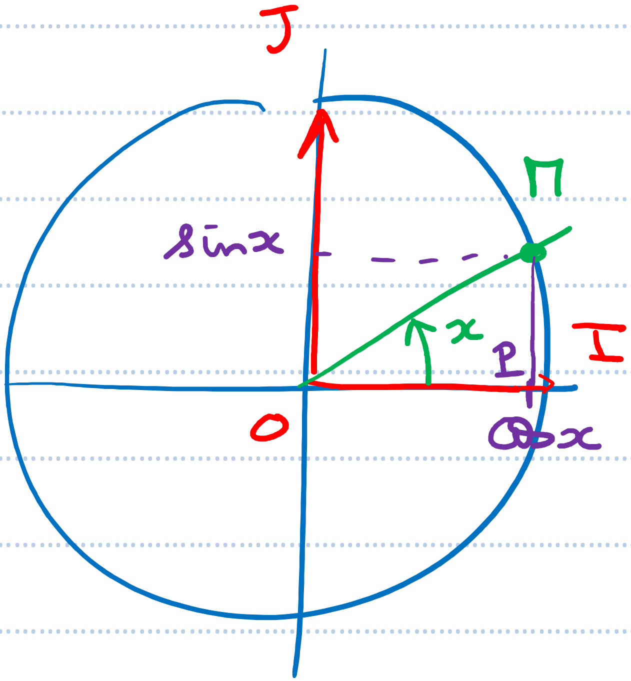
1’

1. $\cos(\mathrm{a}+\mathrm{b}) = \mathrm{cosa}+\mathrm{cosb}$ — 1%
2. $\cos(\mathrm{a}+\mathrm{b}) = \mathrm{sina}.\mathrm{cosb}+\mathrm{sinb}.\mathrm{cosa}$ — 2%
3. $\cos(\mathrm{a}+\mathrm{b}) = \mathrm{sina}.\mathrm{cosb}-\mathrm{sinb}.\mathrm{cosa}$ — 3%
4. $\cos(a+b) = cosa.cosb - sinb.sina$ — 4% ✓4
5. $\cos(a+b) = cosa.cosb + sinb.sina$ — 5%

Laquelle de ces formules est juste ? 1’

1. $\cos^2 a = 1 - \sin^2 a$	1%
2. $\cos^2 a = 1 + \sin^2 a$	2%
3. $\cos^2 a = -1 - \sin^2 a$	3%
4. $\cos^2 a = \sin^2 a - 1$	4%

Notes



Pythagore : $OM^2 = OP^2 + PM^2$

$1 = \cos^2 x + \sin^2 x$

$$\boxed{\cos^2 x = 1 - \sin^2 x}$$

Laquelle de ces formules est juste ?

1’

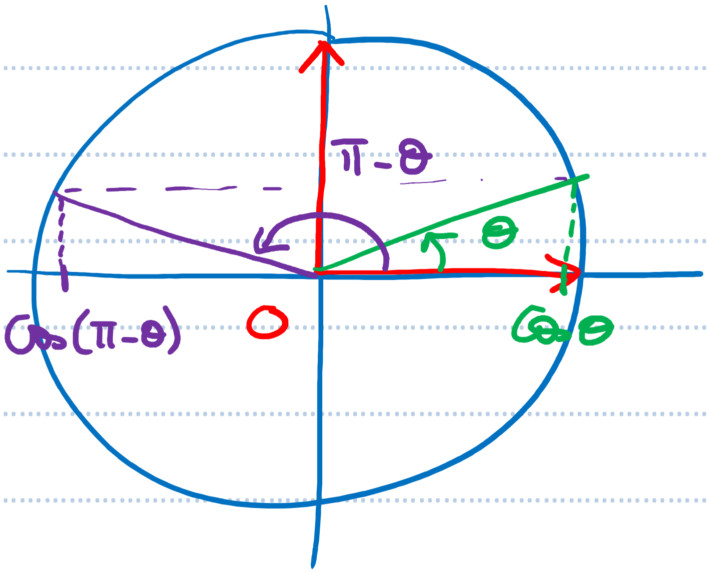
1. $\cos(\pi - \theta) = \cos(\theta)$
2. $\cos(\pi - \theta) = -\cos(\theta)$
3. $\cos(\pi - \theta) = \sin(\theta)$
4. $\cos(\pi - \theta) = -\sin(\theta)$

✓2

1.	1%
2.	2%
3.	3%
4.	4%

Notes

Méth 1



$$\cos(\pi-\theta) = -\cos\theta$$

Méth 2

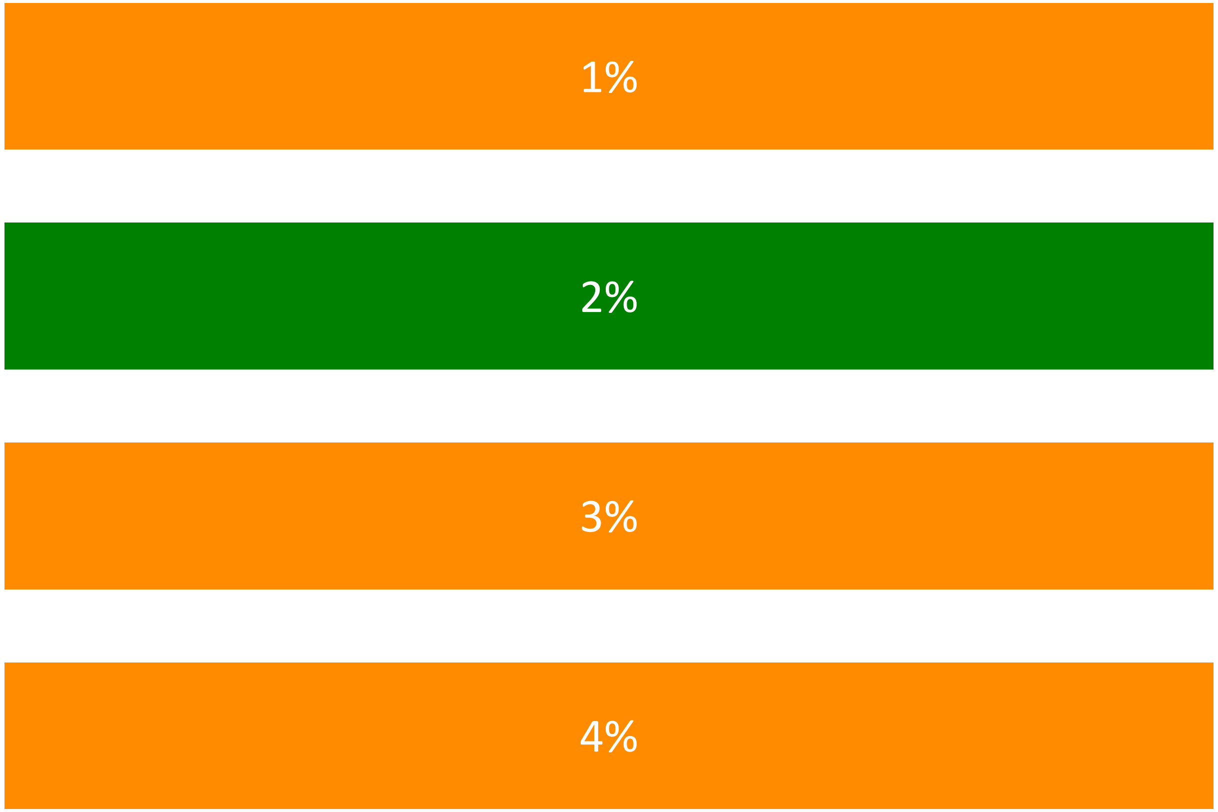
$$\cos(\pi-\theta) = \underbrace{\cos\pi}_{=-1}\cos\theta + \underbrace{\sin\pi}_{=0}\sin\theta = -\cos\theta$$

Q

Quelle est la valeur de cos(2kπ) où $k \in \mathbb{Z}$?

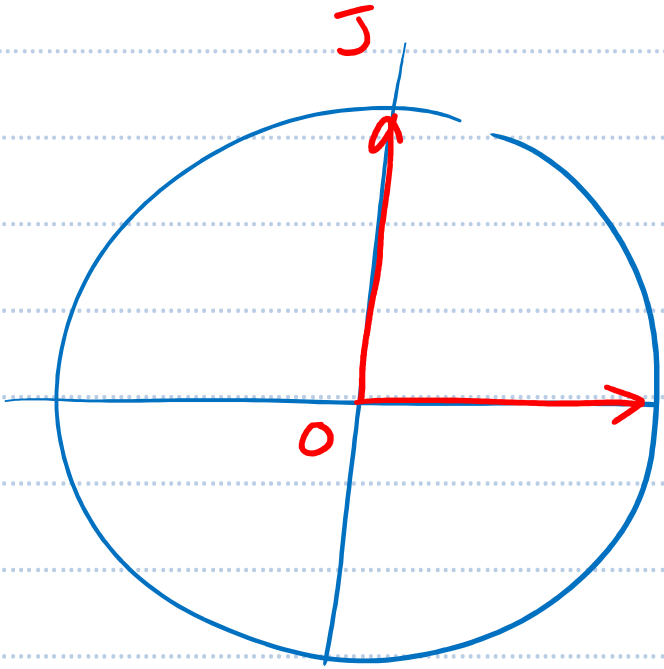
1’

1. 0
2. 1
3. -1
4. $(-1)^k$



$k \in \mathbb{Z}$

Notes



$\underbrace{\dots -3\pi \;;\; 3\pi \;;\; \pi}_{(2k+1)\pi}$

$\underbrace{0,\ 2\pi,\ 4\pi,\ -2\pi,\ -4\pi \text{ etc...}}_{2k\pi}$

$$\cos(2k\pi) = 1 \quad \forall k \in \mathbb{Z}$$

L'ensemble des solutions de l'équation

$\sin(3x) = \frac{\sqrt{2}}{2}$ est :

3'

1. $S = \left\{\frac{\pi}{12} + 2k\pi\ ;\ \frac{3\pi}{12} + 2k\pi\ ;\ k\epsilon\mathbb{Z}\right\}$ — 1%

✓2 2. $S = \left\{\frac{\pi}{12} + \frac{2k\pi}{3};\ \frac{\pi}{4} + \frac{2k\pi}{3};\ k\epsilon\mathbb{Z}\right\}$ — 2%

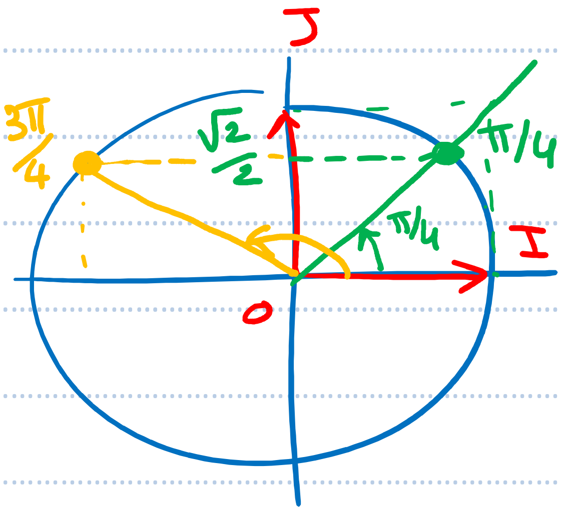
3. $S = \left\{\frac{\pi}{12} + \frac{2k\pi}{3};\ \frac{-\pi}{12} + \frac{2k\pi}{3};\ k\epsilon\mathbb{Z}\right\}$ — 3%

4. $S = \left\{\frac{\pi}{12} + 2k\pi;\ \frac{-\pi}{12} + 2k\pi;\ k\epsilon\mathbb{Z}\right\}$ — 4%

Notes $\sin(3x) = \dfrac{\sqrt{2}}{2}$

On pose $X = 3x$

On résout d'abord $\sin X = \dfrac{\sqrt{2}}{2} \iff X = \dfrac{\pi}{4} + 2k\pi$ ou $\dfrac{3\pi}{4} + 2k\pi$



On revient à x !

$X = 3x = \dfrac{\pi}{4} + 2k\pi$ ou $3x = \dfrac{3\pi}{4} + 2k\pi$

$x = \dfrac{\pi}{12} + \dfrac{2k\pi}{3}$ ou $\dfrac{\pi}{4} + \dfrac{2k\pi}{3}$

$S = \left\{ \dfrac{\pi}{12} + \dfrac{2k\pi}{3} ; \dfrac{\pi}{4} + \dfrac{2k\pi}{3} ; k \in \mathbb{Z} \right\}$

Laquelle de ces formules est juste ?

2'

1. $\sin(2a) = 2sina.cosa$ — 1%
2. $\sin(2a) = 2sina + 2cosa$ — 2%
3. $\sin(2a) = cos^2(a) + sin^2(a)$ — 3%
4. $\sin(2a) = cos^2(a) - sin^2(a)$ — 4%

✓1

Notes

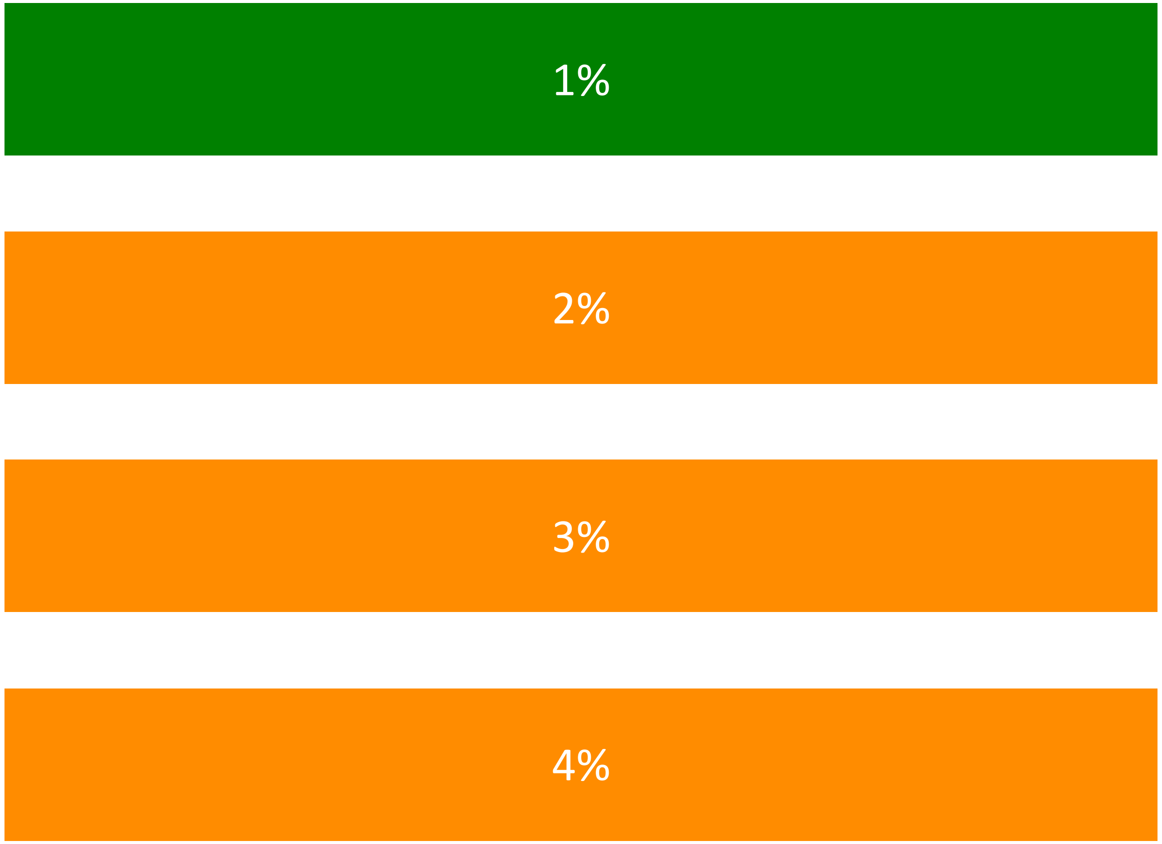
$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a+a) = \sin a . \cos a + \sin a . \cos a$$

$$\sin(2a) = 2 \sin a . \cos a$$

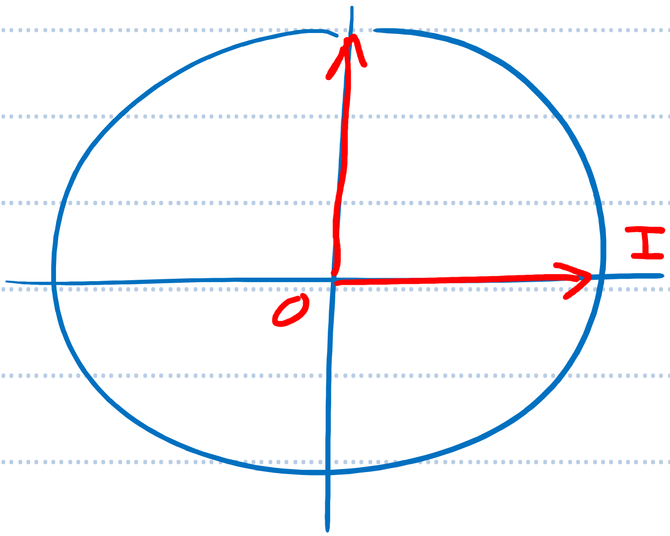
Quelle est la valeur de $\sin(2k\pi)$ où $k \in \mathbb{Z}$?

1. 0
2. 1
3. -1
4. 1/2



Notes

J.



$\leftarrow 2k\pi$

$\sin(2k\pi) = 0 \quad \forall k \in \mathbb{Z}$

La dérivée de cos(5x+7) est :

1’

1. 5.sin(5x+7) — 1%
2. sin(5x+7) — 2%
3. -5sin(5x+7) — 3%
4. $\frac{\sin(5x+7)}{5}$ — 4%
5. $-\frac{\sin(5x+7)}{5}$ — 5%

Notes

$$\big(\underline{\sin}(ax+b)\big)' = a\cos(ax+b)$$

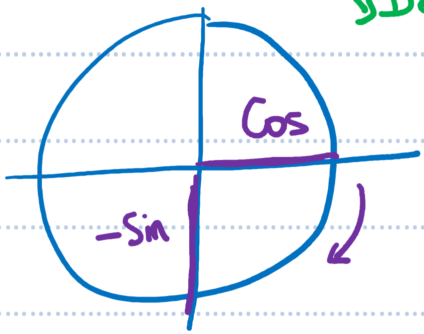
sympa

$$\big(\cos(ax+b)\big)' = -a\sin(ax+b)$$

Donc $\big(\cos(5x+7)\big)' = -5.\sin(5x+7)$.

Droite

Dérivee



Q

Laquelle de ces formules est juste ?

2'

1. $\cos(2a) = 2.cos^2(a) - 1$ — 1%
2. $\cos(2a) = 2sina - 2cosa$ — 2%
3. $\cos(2a) = cos^2(a) + sin^2(a)$ — 3%
4. $\cos(2a) = 1 - 2cos^2(a)$ — 4%

✓1

Notes

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a+a) = \cos a . \cos a - \sin a . \sin a$$

$$\cos(2a) = \cos^2 a - \sin^2 a$$

Comme $\cos^2 a + \sin^2 a = 1$

alors $\sin^2 a = 1 - \cos^2 a$

Ainsi :

$$\cos(2a) = \cos^2 a - (1 - \cos^2 a)$$

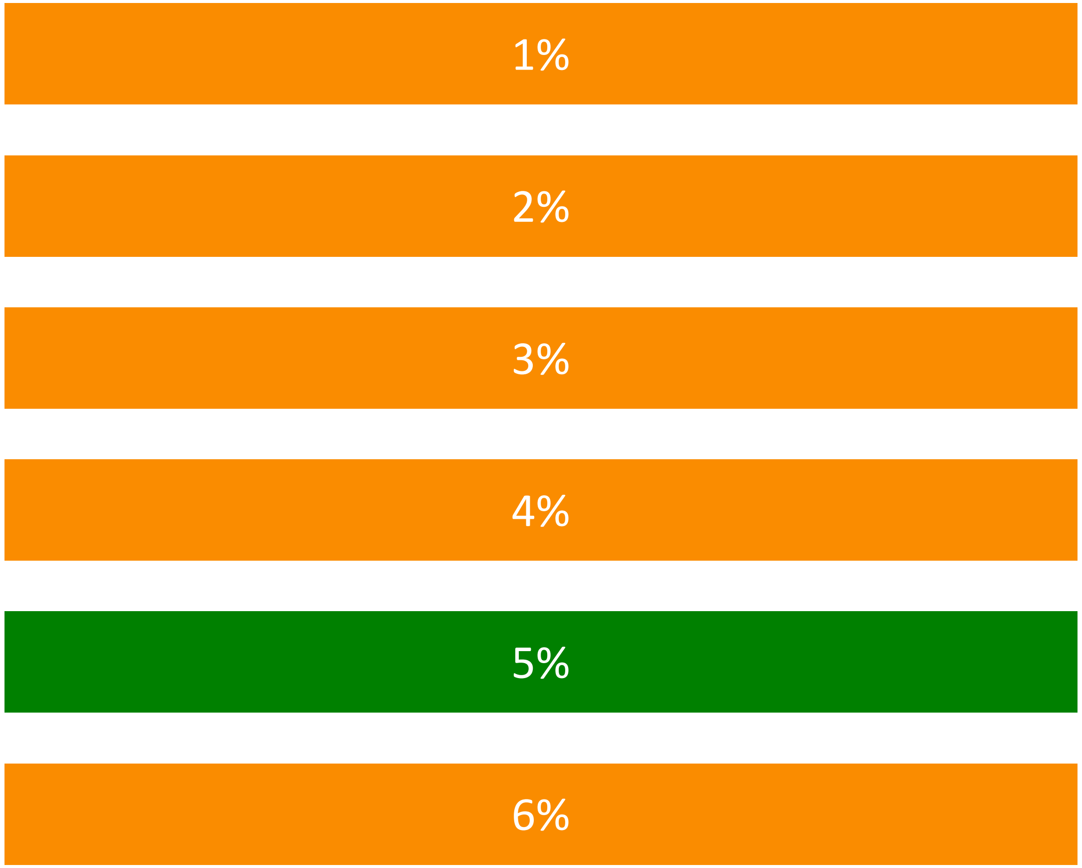
$$= \cos^2 a - 1 + \cos^2 a$$

$$\cos(2a) = 2\cos^2 a - 1$$

Une primitive de cos^2(x) est :

3'

1. $x - \frac{1}{2}\sin(2x)$
2. $\frac{sin^3(x)}{3}$
3. - $\frac{sin^3(x)}{3}$
4. - $\frac{cos^3(x)}{3}$
5. ✓5 $\frac{x}{2} + \frac{1}{4}\sin(2x)$
6. $\frac{cos^3(x)}{3}$



Notes

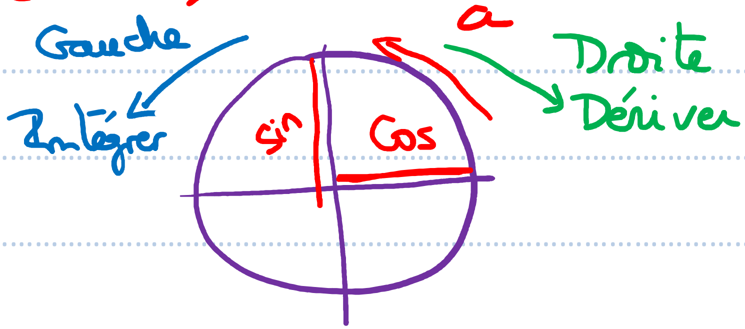
$$\cos(2x) = 2\cos^2 x - 1$$

$$\Longleftrightarrow \quad \cos^2 x = \frac{1+\cos(2x)}{2} = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

Primitive de $\cos^2 x = \frac{1}{2}x + \frac{1}{4}\sin(2x)$

primitives de $\cos(ax+b) = \frac{1}{a}\sin(ax+b) + cte \quad a \neq 0$

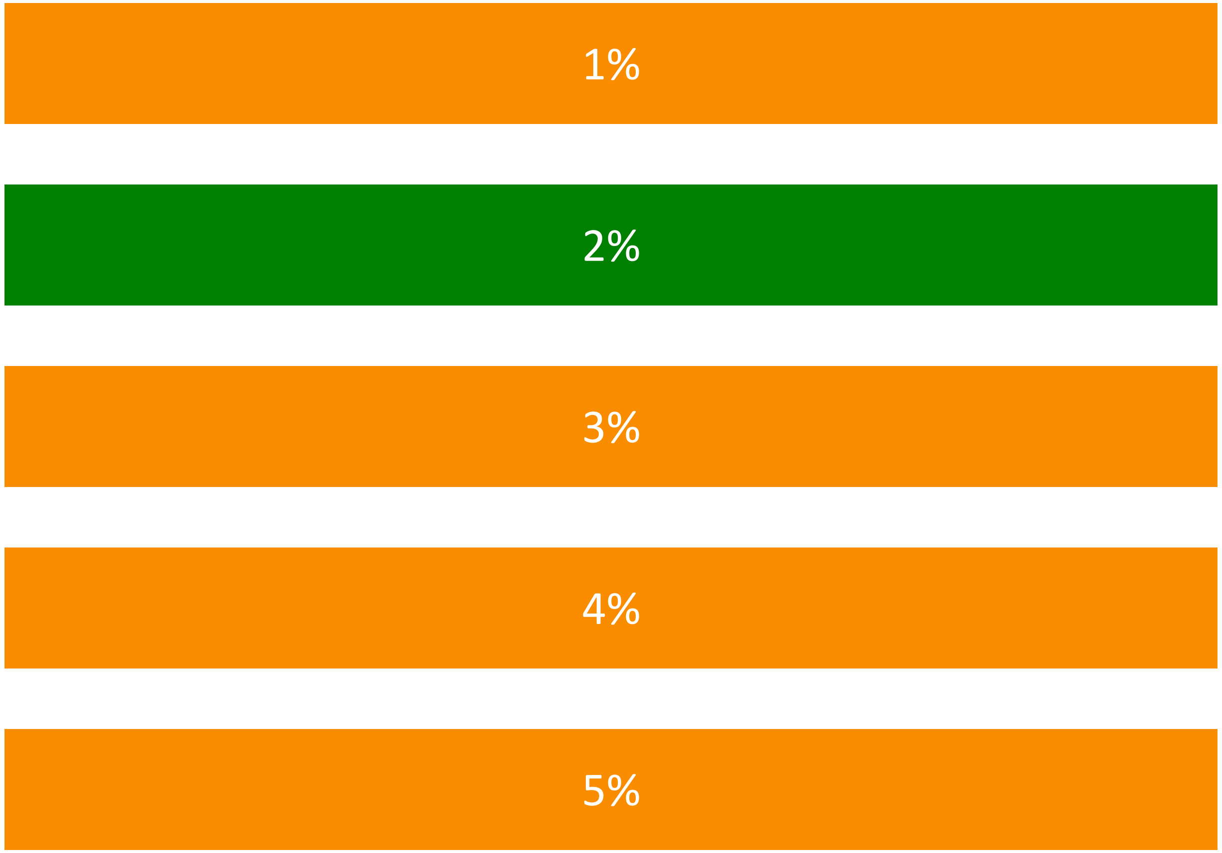
" $\quad \sin(ax+b) = -\frac{1}{a}\cos(ax+b) + cte$



Quelle est la valeur de l'intégrale : $\int_0^{\frac{\pi}{3}} \cos(7x)\,dx$

3'

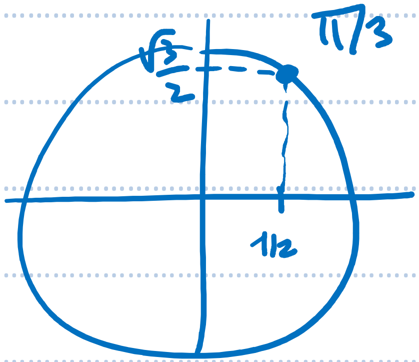
1. $\frac{\sqrt{3}}{2}$
2. $\frac{\sqrt{3}}{14}$
3. $-\frac{1}{2}$
4. $7\frac{\sqrt{3}}{2}$
5. 7/2



Notes

$$I = \int_0^{\pi/3} \cos(7x)\,dx = \left[\frac{1}{7}\sin(7x)\right]_0^{\pi/3}$$

$$I = \frac{1}{7}\left(\underbrace{\sin\left(\frac{7\pi}{3}\right)}_{=\sin(\pi/3)} - \sin 0\right)$$



$$\frac{7\pi}{3} = \underbrace{\frac{6\pi}{3}}_{2\pi} + \frac{\pi}{3}$$

$$I = \frac{1}{7} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{14}.$$

Quel est le module du nombre complexe :
$\underline{Z} = 4 - 3.j$?

1'

1. Z = 5 ✓ 1
2. Z = 7
3. Z = $\sqrt{7}$
4. Z = 25
5. Aucune réponse n'est juste

1.	1%
2.	2%
3.	3%
4.	4%
5.	5%

Notes

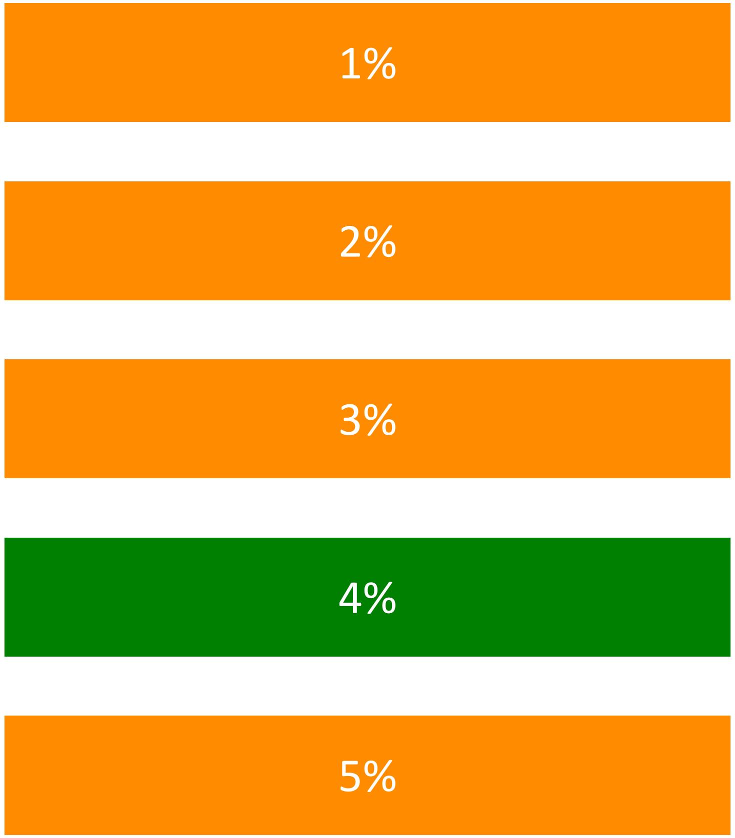
$Z = |4 - 3j| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

Rappel: $|a + jb| = \sqrt{a^2 + b^2}$

Un argument du nombre complexe :

$\underline{Z} = 4 - 3.j$ est :

1. $\arg(\underline{Z}) = \arctan(4/3)$
2. $\arg(\underline{Z}) = \arctan(3/4)$
3. $\arg(\underline{Z}) = \arctan(-3/4)+\pi$
4. $\arg(\underline{Z}) = -\arctan(3/4)$
5. $\arg(\underline{Z}) = \arctan(-4/3)$



Rappel:

Notes

$$\arg(a+jb) = \begin{cases} \arctan\left(\frac{b}{a}\right) & \text{si } a>0 \\ \arctan\left(\frac{b}{a}\right) \pm \pi & \text{si } a<0. \end{cases}$$

$$\arg(jb) = \begin{cases} \pi/2 & \text{si } b>0 \\ -\pi/2 & \text{si } b<0 \end{cases}$$

$$\arg(z) = \arg(\underbrace{4}_{>0} - 3j) = \arctan\left(-\frac{3}{4}\right) = -\arctan\left(\frac{3}{4}\right)$$

Quelle est la partie réelle de : $\underline{Z} = 10.e^{\frac{3j\pi}{4}}$? 2’

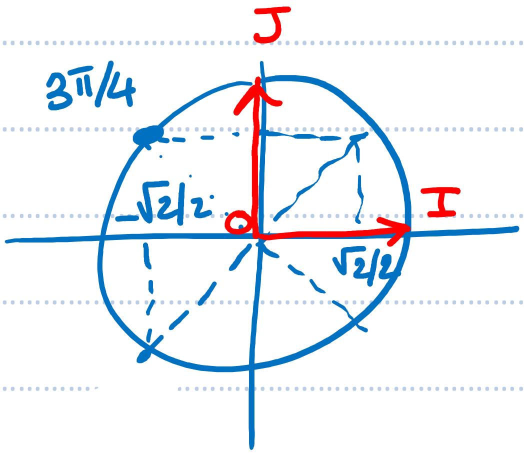
1. $\mathrm{Re}(\underline{Z}) = 10$ — 1%
2. $\mathrm{Re}(\underline{Z}) = 10\frac{\sqrt{2}}{2}$ — 2%
3. ✓3 $\mathrm{Re}(\underline{Z}) = -5\sqrt{2}$ — 3%
4. $\mathrm{Re}(\underline{Z}) = -\frac{\sqrt{2}}{2}$ — 4%
5. $\mathrm{Re}(\underline{Z}) = \frac{\sqrt{2}}{2}$ — 5%

Notes

$$\mathrm{Re}\left(10\, e^{3j\pi/4}\right) = ?$$

Rappel : $\underline{Z} = Z e^{j\theta} = Z(\cos\theta + j\sin\theta) = \underbrace{Z\cos\theta}_{\mathrm{Re}(\underline{Z})} + j\underbrace{Z\sin\theta}_{\mathrm{Im}(\underline{Z})}$

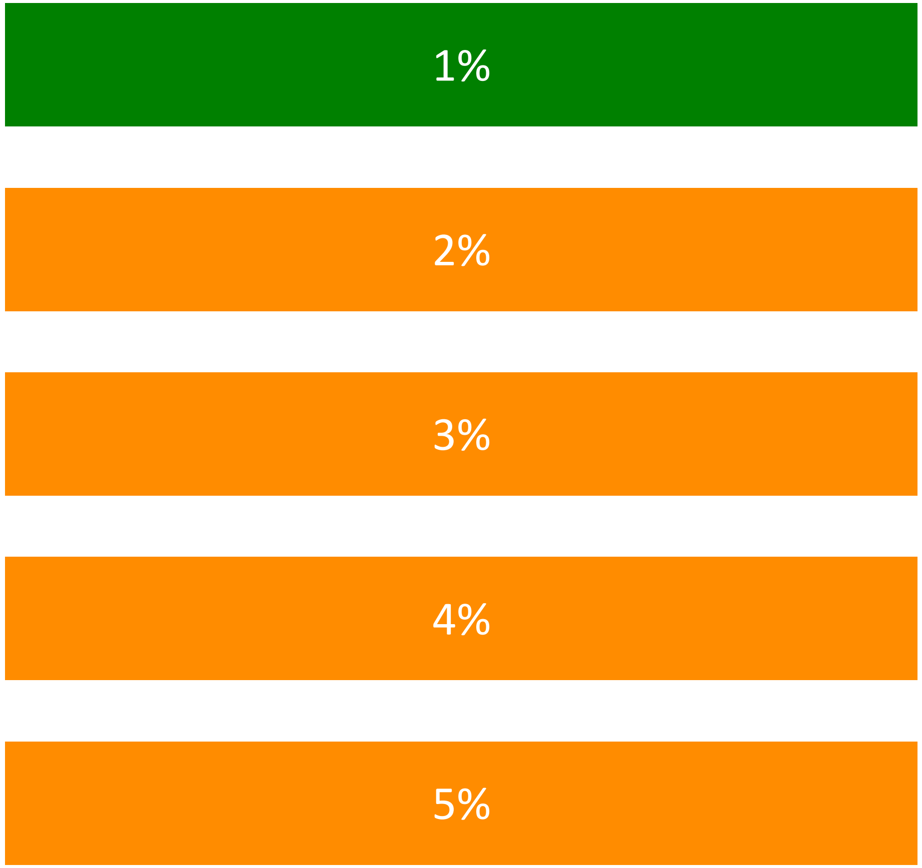
$$\mathrm{Re}\left(10\, e^{3j\pi/4}\right) = 10 \cdot \cos\left(\frac{3\pi}{4}\right) = -10\frac{\sqrt{2}}{2} = -5\sqrt{2}$$



Quelle est la partie imaginaire de : $\underline{Z} = 10.e^{\frac{3j\pi}{4}}$?

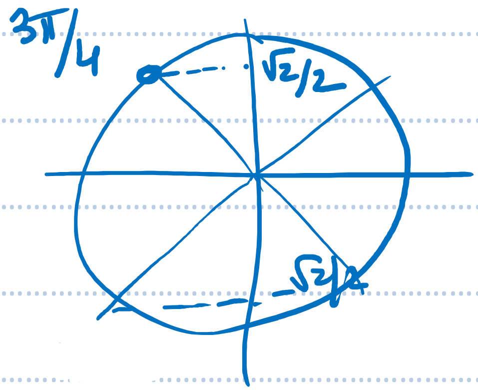
2’

1. $\mathrm{Im}(\underline{Z}) = 10\frac{\sqrt{2}}{2}$
2. $\mathrm{Im}(\underline{Z}) = \frac{3\pi}{4}$
3. $\mathrm{Im}(\underline{Z}) = -5\sqrt{2}$
4. $\mathrm{Im}(\underline{Z}) = -\frac{\sqrt{2}}{2}$
5. $\mathrm{Im}(\underline{Z}) = \frac{\sqrt{2}}{2}$



Notes

$$\mathrm{Im}\left(10e^{3j\pi/4}\right) = 10\sin\left(\frac{3\pi}{4}\right) = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$



Le conjugué de $\underline{Z} = 10.e^{\frac{3j\pi}{4}}$ est :

1'

1. $\underline{Z}^* = 5\sqrt{2}(-1+j)$ — 1%
2. $\underline{Z}^* = 5\sqrt{2}(-1-j)$ — 2% ✓2
3. $\underline{Z}^* = 5\sqrt{2}(1+j)$ — 3%
4. $\underline{Z}^* = 5\sqrt{2}(1-j)$ — 4%
5. $\underline{Z}^* = 10.e^{\frac{3j\pi}{4}}$ — 5%

$$\underline{z} = 10 \cdot e^{3j\frac{\pi}{4}} = -5\sqrt{2} + j\,5\sqrt{2}$$

Notes

Conjugué de $\underline{z}$: $\underline{z}^* = -5\sqrt{2} - j\,5\sqrt{2} = 10 \cdot e^{-3j\pi/4}$

Rappel : $\underline{z} = z e^{j\theta} = a + jb$

$$\underline{z}^* = z e^{-j\theta} = a - jb$$

Q

Quelle est la forme algébrique de : $\underline{Z} = e^{\frac{-j\pi}{2}}$? 2’

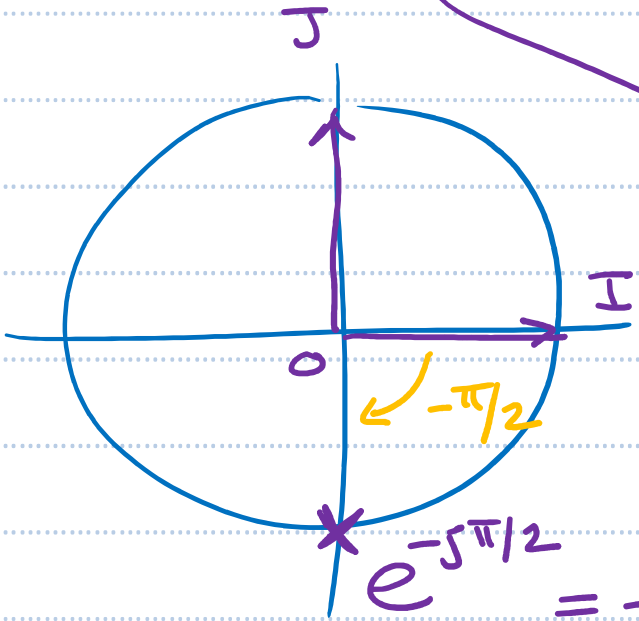
1. $\underline{Z}$ = - j	1%
2. $\underline{Z}$ = 1+ j	2%
3. $\underline{Z}$ = 1− j	3%
4. $\underline{Z}$ = j	4%
5. *Aucune réponse n'est la bonne.*	5%

✓1

Notes

$$z = \underbrace{e^{-j\pi/2}} = \underbrace{\cos\left(-\pi/2\right)}_{=0} + j\,\underbrace{\sin\left(-\pi/2\right)}_{-1} = -j$$

$\rightarrow$ graphiquement :



$$e^{-j\pi/2} = -j$$

$$-1 = ?$$

Quel est le module du nombre complexe :

$\underline{Z} = -7.j.e^{\frac{j\pi}{9}}$?

2’

1. Z = -7	1%
2. Z = 7j	2%
3. Z =-7j	3%
4. Z = 7	4%
5. Aucune réponse n’est juste	5%

Notes

$$Z = -7j\,e^{j\pi/9} = 7 \times -j \times e^{j\pi/9}$$

Méthode 1

$$Z = \underbrace{7}_{\text{module}} \cdot e^{-j\pi/2} \cdot e^{j\pi/9}$$

Méthode 2

$$Z = \left|-7j\,e^{j\pi/9}\right| = \underbrace{|-7j|}_{=7} \times \underbrace{\left|e^{j\pi/9}\right|}_{=1} = 7$$

Remarque : $|e^{j\theta}| = 1$

en effet $|e^{j\theta}| = |\cos\theta + j\sin\theta| = \sqrt{\cos^2\theta + \sin^2\theta}$
$= \sqrt{1} = 1$.

Un argument du nombre complexe : $\underline{Z} = -7.j.e^{\frac{j\pi}{9}}$ est :

3’

1. $\arg(\underline{Z}) = \frac{\pi}{9}$ — 1%
2. $\arg(\underline{Z}) = \frac{-7\pi}{9}$ — 2%
3. $\arg(\underline{Z}) = \frac{11}{18}$ — 3%
4. *Aucune réponse n'est la bonne* — 4%
5. $\arg(\underline{Z}) = -\frac{7\pi}{18}$ — 5%

✓5

Méthode 1

$$\underline{z} = -7j\, e^{j\pi/9}$$

Notes

$$\underline{z} = 7\, e^{-j\pi/2}\, e^{j\pi/9}$$

$$\underline{z} = 7\, e^{j\left(-\frac{\pi}{2}+\frac{\pi}{9}\right)} = 7\, e^{j\,\frac{-9\pi+2\pi}{18}}$$

$$\underline{z} = 7\, e^{-j\frac{7\pi}{18}}$$

$$|\underline{z}| = 7$$

$$\arg(\underline{z}) = -\frac{7\pi}{18}$$

Méthode 2

$$\arg(\underline{z}) = \arg(-7j) + \arg\left(e^{j\pi/9}\right)$$

$$= -\frac{\pi}{2} + \frac{\pi}{9}$$

$$\arg(\underline{z}) = -\frac{7\pi}{18}$$

Le conjugué du nombre complexe : $\underline{Z} = -7.\mathrm{j}.e^{\frac{j\pi}{9}}$ est :

1’

1. $\underline{Z}^* = 7.e^{\frac{j\pi}{9}}$ — 1%
2. $\underline{Z}^* = 7.e^{\frac{-j\pi}{9}}$ — 2%
3. $\underline{Z}^* = 7.e^{\frac{7j\pi}{18}}$ — 3%
4. $Aucune\ réponse\ n'est\ la\ bonne$ — 4%

$$Z = -7j \cdot e^{j\frac{\pi}{9}}$$

Notes

$Z = 7$ et $\arg(Z) = \frac{-7\pi}{18}$

Donc $Z = 7 \cdot e^{-7j\pi/18}$

et

$$Z^* = 7 \cdot e^{7j\pi/18}$$

Q

ω est un réel. Quelle est le module du nombre complexe suivant ?

$$T(\omega) = \frac{1 + 2j\omega}{1 - j\omega}$$

3'

1. $|T(\omega)| = \dfrac{\sqrt{1 + 4\omega^2}}{\sqrt{1 + \omega^2}}$ ✓1 — 1%
2. $|T(\omega)| = \dfrac{\sqrt{5}}{\sqrt{2}}$ — 2%
3. $|T(\omega)| = \sqrt{5} - \sqrt{2}$ — 3%
4. $|T(\omega)| = \dfrac{\sqrt{1 + 2\omega^2}}{\sqrt{1 + \omega^2}}$ — 4%
5. $|T(\omega)| = \sqrt{1 + 4\omega^2} - \sqrt{1 + \omega^2}$ — 5%

Notes

$$T(\omega) = \frac{1 + 2j\omega}{1 - j\omega}$$

$$|T(\omega)| = \left| \frac{1 + 2j\omega}{1 - j\omega} \right| = \frac{|1 + 2j\omega|}{|1 - j\omega|}$$

$$|T(\omega)| = \frac{\sqrt{1^2 + (2\omega)^2}}{\sqrt{1^2 + \omega^2}} = \frac{\sqrt{1 + 4\omega^2}}{\sqrt{1 + \omega^2}}$$

Q

Déterminer un argument du nombre complexe suivant : $$T(\omega) = \frac{1+2j\omega}{1-j\omega}$$

2'

1. $\mathrm{Arg}(T(\omega)) = \arctan(2\omega) - \arctan(\omega)$ — 1%
2. $\mathrm{Arg}(T(\omega)) = \dfrac{\arctan(2\omega)}{\arctan(\omega)}$ — 2%
3. $\mathrm{Arg}(T(\omega)) = \arctan(2\omega) + \arctan(\omega)$ — 3% ✓3
4. $\mathrm{Arg}(T(\omega)) = \arctan\left(\dfrac{1}{2\omega}\right) + \arctan\left(\dfrac{1}{\omega}\right)$ — 4%
5. $\mathrm{Arg}(T(\omega)) = \arctan\left(\dfrac{1}{2\omega}\right) - \arctan\left(\dfrac{1}{\omega}\right)$ — 5%

Notes

$$T(\omega) = \frac{1+2j\omega}{1-j\omega}$$

$$\arg\left(T(\omega)\right) = \arg(1+2j\omega) - \arg(1-j\omega)$$

$$= \arctan\left(\frac{2\omega}{1}\right) - \arctan\left(\frac{-\omega}{1}\right)$$

$$\arg\left(T(\omega)\right) = \arctan(2\omega) + \arctan\omega$$

Simplifier le nombre complexe suivant :

$$\underline{Z} = (-2 + 2j\sqrt{3})^{2025}$$

4'

1. $\underline{Z} = 2^{2025}$
2. $\underline{Z} = -2^{2025}$
3. $\underline{Z} = j2^{2025}$
4. $\underline{Z} = j4^{2025}$
5. Aucune des réponses citées n'est juste

1.	1%
2.	2%
3.	3%
4.	4%
5.	5%

$$Z = \left(-2+2j\sqrt{3}\right)^{2025}.$$

Notes

$$* \ Z = \left|-2+2j\sqrt{3}\right|^{2025} = \sqrt{\underbrace{4+4\times 3}_{16}}^{\,2025} = 4^{2025}$$

$$* \ \arg(Z) = 2025.\arg\left(-2+2j\sqrt{3}\right) = 2025\left(\arctan\left(\frac{2\sqrt{3}}{-2}\right)+\pi\right)$$

$$= 2025.\left(\arctan(-\sqrt{3})+\pi\right)$$

$$= 2025.\left(-\frac{\pi}{3}+\pi\right) = \frac{2\pi}{3}\times 2025 = 2\times 675\pi$$

$$\arg(Z) = 0$$

$$\text{Ainsi} \quad Z = 4^{2025}\, e^{j.0} = 4^{2025}$$

C et ω sont des réels. Déterminer un argument du nombre complexe suivant : $(1+jC\omega)^2$

2’

1. $(\arctan(C\omega))^2$ — 1%
2. $(\arctan(1/C\omega))^2$ — 2%
3. ✓3 $2\arctan(C\omega)$ — 3%
4. Arctan(2 $C\omega$) — 4%
5. Aucune réponse n’est juste — 5%

$$\arg\left((1+jC\omega)^2\right) = 2\arg(1+jC\omega)$$

Notes

$$\arg\left((1+jC\omega)^2\right) = 2\arctan(C\omega)$$

Remarque : $\left|(1+jC\omega)^2\right| = |1+jC\omega|^2 = \sqrt{1+C^2\omega^2}^{\,2} = 1+C^2\omega^2$.