Corrigé du DM20

Exercice 3 p19 :

$$P = \int_0^1 \frac{dx}{x^2 - 4x + 5}$$

on pose $t = x - 2$

$x = t + 2$

Bornes : $\begin{cases} x = 0 & t = -2 \\ x = 1 & t = -1 \end{cases}$

Relation dx et dt

$$(x-2)' = 1 = \frac{dt}{dx} \Rightarrow dt = dx$$

$$P = \int_{-2}^{-1} \frac{dt}{\underbrace{(t+2)^2 - 4(t+2) + 5}_{t^2 + 4t + 4 - 4t - 8 + 5}} = \frac{dt}{t^2 + 1}$$

$$P = \left[\arctan(t)\right]_{-2}^{-1} = \arctan(-1) - \arctan(-2)$$

$$P = -\frac{\pi}{4} - \arctan(-2)$$

$$I = \int_0^{\pi/2} \frac{\cos(t)}{(1+\sin(t))^2}\,dt \quad \text{avec } x = 1+\sin(t)$$

bornes $\begin{cases} \cancel{t} = 0 \\ t = \frac{\pi}{2} \end{cases} \iff \begin{cases} x = 1 \\ x = 2 \end{cases}$

$\frac{dx}{dt} = (1+\sin t)' = \cos t$

$dx = \cos t \, dt$

$$I = \int_1^2 \frac{dx}{x^2} = \left[-x^{-1}\right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$

$$I = \int_1^2 \frac{dx}{x^2} = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$k = \int_1^e \frac{\ln^3(t) + \ln^2(t) + 1}{t}\, dt$$

Bornes : $\begin{cases} t = e \\ t = 1 \end{cases} \Rightarrow \begin{cases} x = 1 \\ x = 0 \end{cases}$

Relation dx et dt :

$$\frac{dx}{dt} = \frac{1}{t} \Rightarrow dx = \frac{dt}{t}$$

$$K = \int_0^1 \frac{x^3 + x^2 + 1}{1}\, dx = \left[\frac{x^4}{4} + \frac{x^3}{3} + x\right]_0^1$$

$$K = \frac{1}{4} + \frac{1}{3} + 1 = \frac{19}{12}$$

$$L = \int_0^1 \frac{dx}{(x^2+1)^2} \quad \text{avec} \quad x = \tan(t)$$

bornes $\begin{cases} x = 0 \\ x = 1 \end{cases} \iff \begin{cases} t = 0 \\ t = \frac{\pi}{4} \end{cases}$

$$\frac{dx}{dt} = (1+\tan^2 t)$$

$$L = \int_0^{\frac{\pi}{4}} \frac{(1+\tan^2 t)\,dt}{(1+\tan^2 t)^2} = \int_0^{\pi/4} \frac{dt}{1+\tan^2 t}$$

or $1+\tan^2 t = \dfrac{1}{\cos^2 t}$

donc $L = \displaystyle\int_0^{\pi/4} \cos^2 t\,dt = \int_0^{\pi/4} \frac{1+\cos(2t)}{2}\,dt$

$$L = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1+\cos(2t))\,dt = \frac{1}{2}\left[t + \frac{\sin(2t)}{2}\right]_0^{\pi/4}$$

$$L = \frac{1}{2}\left(\frac{\pi}{4} + \frac{\sin(\pi/2)}{2}\right) = \frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right) = \frac{1}{4}\left(\frac{\pi}{2}+1\right)$$