Exercice 1 (5,5)

- $f(k) = \cos(k + \frac{\pi}{3}).U(k)$

$f(k) = (\frac{1}{2}\cos(k) - \frac{\sqrt{3}}{2}\sin(k)).\mathcal{U}(k)$

$TZ[f(k)] = F(z) = \frac{1}{2} TZ[\cos k] - \frac{\sqrt{3}}{2} TZ[\sin(k)]$

$$F(z) = \frac{1}{2}\frac{z^2 - z\cos(1)}{z^2 - 2z\cos(1) + 1} - \frac{\sqrt{3}}{2}\frac{z\sin(1)}{z^2 - 2z\cos(1) + 1}$$

$$\boxed{F(z) = \frac{1}{2} \cdot \frac{z^2 - z(\cos(1) + \sqrt{3}\sin(1))}{z^2 - 2z\cos(1) + 1}}$$

- $g(k) = k^2.\mathcal{U}(k) = k.\underbrace{k.\mathcal{U}(k)}_{m(k)}$

$TZ[g(k)] = G(z) = -z.\Pi'(z)$

$\rightarrow \Pi(z) = -z.TZ[\mathcal{U}(k)]' = -z.\left(\frac{z}{z-1}\right)' = \frac{z}{(z-1)^2}$

$$\rightarrow \Pi'(z) = \frac{(z-1)^2 - z\,2(z-1)}{(z-1)^4}$$

$$\rightarrow G(z) = -z.\frac{z^2 - 2z + 1 - 2z^2 + 2z}{(z-1)^4}$$

$$\boxed{G(z) = \frac{z^3 - z}{(z-1)^4}}$$

③ $h(k) = (k^2 + 3k - 1)\,\mathcal{U}(k-3)$ soit $k = k+3$

alors $h(k) = (k+3)^2\,\mathcal{U}(k) + 3(k+3)\,\mathcal{U}(k) - \mathcal{U}(k)$

$= k^2\,\mathcal{U}(k) + 6k\,\mathcal{U}(k) + 18\,\mathcal{U}(k) + \underbrace{3k\,\mathcal{U}(k)}_{9\mathcal{U}(k)} - \mathcal{U}(k)$

$= k^2\,\mathcal{U}(k) + 9k\,\mathcal{U}(k) + 17\,\mathcal{U}(k)$

$$H(z) = \frac{z(z+1)}{(z-1)^3} + 9\,\frac{z}{(z-1)^2} + 17 \times \frac{z}{z-1}$$

On applique à nouveau le retard alors on obtient :

$$H(z) = z^{-3}\left(\frac{z(z+1)}{(z-1)^3} + 9\,\frac{z}{(z-1)^2} + 17\,\frac{z}{z-1}\right)$$

$$H(z) = \frac{z+1}{z^2(z-1)^3} + \frac{9}{z^2(z-1)^2} + 17\,\frac{1}{z^2(z-1)}.$$

Exercice 2. 2

$$F(z) = \frac{z}{z^2 - z\sqrt{2} + 2}$$

$\Delta = b^2 - 4ac = (-\sqrt{2})^2 - 4 \times 1 \times 2 = -6 < 0$

donc formulaire à utiliser

On identifie : $a^2 = 2$ donc $a = \sqrt{2}$.

on a donc au dénominateur :

$-2 \times \sqrt{2} \times \cos(\omega) = -\sqrt{2} \Leftrightarrow \cos(\omega) = \frac{-\sqrt{2}}{-2\sqrt{2}} = \frac{1}{2} \Leftrightarrow \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$$F(z) = \frac{a z \sin(\omega)}{z^2 - 2az\cos(\omega) + a^2} = \frac{z . \sqrt{2} . \sin\left(\frac{\pi}{3}\right)}{z^2 - z . 2 . \sqrt{2} . \cos\left(\frac{\pi}{3}\right) + \sqrt{2}^2}$$

$$F(z) = \frac{2}{\sqrt{6}} \frac{z \frac{\sqrt{6}}{2}}{z^2 - z\sqrt{2} + 2} = F(z) = \frac{z}{z^2 - 2\sqrt{2} + 2}$$

Tz^{-1}

$f(k) = a^k . \sin(\omega k) . \mathcal{U}(k) = \frac{2}{\sqrt{6}} \cdot \sqrt{2}^k \cdot \sin\left(\frac{\pi}{3} . k\right) . \mathcal{U}(k)$

TB.

Exercice 3. 3,5 $3y(k) + y(k-1) = \delta(k-2)$.

Tz

$3Y(z) + z^{-1}Y(z) = z^{-2}$

$\Leftrightarrow Y(z)\left(3 + z^{-1}\right) = z^{-2} \Leftrightarrow Y(z) = \frac{z^{-2} \times z}{z^{-1} + 3 \times z}$

$\Leftrightarrow Y(z) = z^{-1} . \frac{1}{3z + 1} = z^{-1} . \frac{1}{3} . \frac{\cancel{z}}{z + \frac{1}{3}}$

$$Y(z) = z^{-2} \cdot \frac{1}{3} \cdot \frac{z}{z+\frac{1}{3}}$$

T_z^{-1}

$$y(k) = \frac{1}{3} \times \left(-\frac{1}{3}\right)^{k-2} \times u(k-2)$$

Exercice 4. 9,5

① $y(k) + 2y(k-1) - 3y(k-2) = x(k-1)$

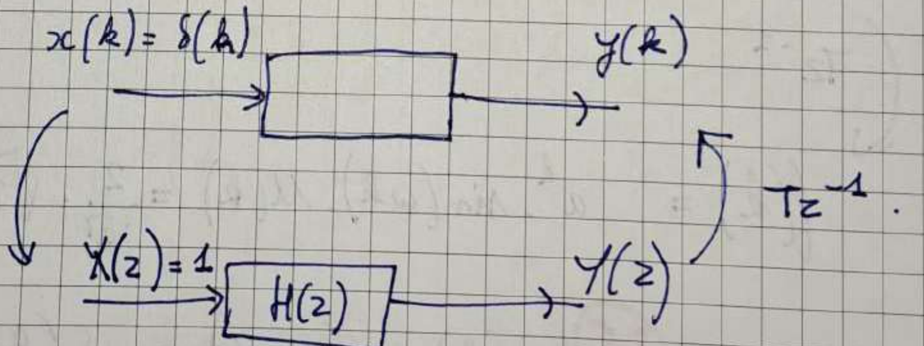
T_z

$$Y(z) + z^{-1} 2 \cdot Y(z) - z^{-2} 3 \cdot Y(z) = z^{-1} \cdot X(z).$$

$$\Leftrightarrow Y(z)\left(1 + 2z^{-1} - 3 \cdot z^{-2}\right) = z^{-1} \cdot X(z).$$

$$\Leftrightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} \times z^2}{\left(-3z^{-2} + 2z^{-1} + 1\right) \times z^2} \Leftrightarrow H(z) = \frac{z}{z^2 + 2z - 3}$$

② Avec $x = \delta(k)$:



$$Y(z) = X(z) \cdot H(z) = 1 \times \frac{z}{z^2+2z-3} = \frac{z}{z^2+2z-3}.$$

$$Y(z) = z \cdot G(z) = z \cdot \frac{1}{\underbrace{z^2+2z-3}}.$$

$\Delta = b^2 - 4ac = 2^2 - 4 \times 1 \times -3 = 16$

DSES :

$$x_1 = \frac{-b+\sqrt{\Delta'}}{2a} = \frac{-2+4}{2} = \frac{2}{2} = 1.$$

$$x_2 = \frac{-b-\sqrt{\Delta'}}{2a} = \frac{-2-4}{2} = -3.$$

$$\hookrightarrow G(z) = \frac{1}{(z-1)(z+3)} = \frac{a}{z-1} + \frac{b}{z+3}.$$

$$a = \left[G(z).(z-1)\right]_{z=1} = \frac{1}{z+3} = \frac{1}{4}.$$

$$b = \left[G(z).(z+3)\right]_{z=-3} = \frac{1}{z-1} = -\frac{1}{4}.$$

$$Y(z) = \frac{1}{4} \cdot \frac{z}{z-1} - \frac{1}{4} \frac{z}{z+3}.$$

Tz^{-1}

$$y(k) = \frac{1}{4} . u(k) - \frac{1}{4} . (-3)^k . u(k).$$

$$y(k) = \frac{1}{4} . u(k)\left(1 - (-3)^k\right).$$

$y(k)$ est la réponse impulsionnelle du système.

3 Avec $x(k) = 2^k . u(k-1)$.

on pose $f(k) = x(k+1) = 2^{k+1} . u(k) = 2^k \times 2^1 . u(k)$.

donc $X(z) = Tz\, f(k-1) = z^{-1} . Tz(f(k)) = 2z^{-1} . \frac{z}{z-2}$

$$X(z) = \frac{2}{z-2}.$$

$$Y(z) = \frac{2}{z-2} \times \frac{z}{z^2+2z-3} = 2 \cdot \frac{1}{z-2} \cdot \frac{z}{(z-1)(z+3)}$$

$$Y(z) = 2 \cdot \frac{1}{z-2} \cdot \left(\frac{1}{4} \times \frac{z}{z-1} - \frac{1}{4} \cdot \frac{z}{z+3} \right)$$

$$Y(z) = \frac{1}{2} \times \left(\frac{z}{(z-2)(z-1)} - \frac{z}{(z-2)(z+3)} \right)$$

$$Y(z) = \frac{z}{2} \times \left(\underbrace{\frac{1}{(z-2)(z-1)}}_{g(z)} - \underbrace{\frac{1}{(z-2)(z+3)}}_{j(z)} \right)$$

DSES

- $g(z) = \frac{a}{z-2} + \frac{b}{z-1} \Rightarrow a = \left[g(z) \cdot (z-2)\right]_{z=2} = 1$

$$b = \left[g(z) \cdot (z-1)\right]_{z=1} = -1.$$

$$g(z) = \frac{1}{z-2} - \frac{1}{z-1}.$$

- $j(z) = \frac{a}{(z-2)} + \frac{b}{(z+3)} \Rightarrow a = \left[j(z) \cdot (z-2)\right]_{z=2} = \frac{1}{5}$

$$b = \left[j(z) \cdot (z+3)\right]_{z=-3} = -\frac{1}{5}.$$

$$j(z) = \frac{1}{5} \left(\frac{1}{z-2} - \frac{1}{z+3} \right).$$

$$Y(z) = \frac{1}{2} \left(\frac{z}{z-2} - \frac{z}{z-1} - \frac{1}{5} \left(\frac{z}{z-2} - \frac{z}{z+3} \right) \right)$$

TZ^{-1}

$$y(k) = \frac{1}{2} \left(2^k \cdot u(k) - 1^k \cdot u(k) - \frac{1}{5} \cdot 2^k \cdot u(k) + \frac{1}{5} (-3)^k u(k) \right)$$

$$y(k) = \frac{1}{2} \cdot u(k) \left(2^k - 1^k - \frac{2^k}{5} + \frac{(-3)^k}{5} \right)$$

$y(k)$ est la réponse du système quand $x(k) = 2^k u(k-1)$.

TB