Correction Ex. Supplémentaires
Partie Trigonométrie

P.24

Ex 6 $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\,\frac{1}{2}$

$\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2}}{4}(\sqrt{3} - 1) > 0$

Si on élève au carré ce résultat, on obtient :

$\sin^2\left(\frac{\pi}{12}\right) = \frac{2}{16}\left(3 - 2\sqrt{3} + 1\right) = \frac{1}{8}\left(4 - 2\sqrt{3}\right) = \frac{1}{4}\left(2 - \sqrt{3}\right)$

Donc $\sin\left(\frac{\pi}{2}\right) = \frac{1}{2}\sqrt{2 - \sqrt{3}}$ soit le résultat du ①

Ex 7

① $E = \cos(2a) = \cos^2 a - \sin^2 a \neq \cos a - \sin a$

② $E = \sqrt{2}\sin\left(\frac{\pi}{4} - a\right) = \sqrt{2}\left(\sin\frac{\pi}{4}\cos a - \sin a\cos\frac{\pi}{4}\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos a - \frac{\sqrt{2}}{2}\sin a\right)$

$E = \cos a - \sin a$ ⟵ c'est donc la réponse cherchée.

③ $E = \sin(2a) = 2\sin a\cos a \neq \cos a - \sin a$

Ex 8

① $\cos(a+b) - \cos(a-b) = -2\sin a\sin b.$

On remplace $a = \frac{p+q}{2}$ et $b = \frac{p-q}{2}$

alors $a+b = p$ et $a-b = q$

d'où la formule : $\cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$

② $\underbrace{\cos x - \cos 2x} = \sin\left(\frac{3x}{2}\right)$ (E)

$p = x \quad q = 2x$

alors $\frac{p+q}{2} = \frac{3x}{2}$ et $\frac{p-q}{2} = -\frac{x}{2}$

L'équation (E) devient alors :

$$-2\sin\left(\frac{3x}{2}\right)\cdot\underbrace{\sin\left(-\frac{x}{2}\right)}_{=-\sin\left(\frac{x}{2}\right)} = \sin\left(\frac{3x}{2}\right)$$

$$\Leftrightarrow \sin\left(\frac{3x}{2}\right) - 2\sin\left(\frac{3x}{2}\right)\sin\frac{x}{2} = 0$$

$$\Leftrightarrow \sin\left(\frac{3x}{2}\right)\left(1 - 2\sin\frac{x}{2}\right) = 0$$

$$\Leftrightarrow \sin\left(\frac{3x}{2}\right) = 0 \quad \text{ou} \quad \sin\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$\Leftrightarrow \frac{3x}{2} = k\pi \quad \text{ou} \quad \frac{x}{2} = \frac{\pi}{6} + 2k\pi \quad \text{ou} \quad \frac{x}{2} = \frac{5\pi}{6} + 2k\pi$$

$$\Leftrightarrow x = \frac{2k\pi}{3} \quad \text{ou} \quad x = \frac{\pi}{3} + 4k\pi \quad \text{ou} \quad x = \frac{5\pi}{3} + 4k\pi$$

$$S = \left\{ \frac{2k\pi}{3} \,;\, \frac{\pi}{3} + 4k\pi \,;\, \frac{5\pi}{3} + 4k\pi \,;\, k \in \mathbb{Z} \right\}.$$

Ex 9

1) $A\cos(\omega t+\varphi) = A\cos\omega t\cos\varphi - A\sin\omega t\sin\varphi$

2) $f(t) = \underbrace{a\cos\omega t}+\underbrace{b\sin\omega t} = \underbrace{A\cos\omega t\cos\varphi}\underbrace{-A\sin\omega t\sin\varphi}$

$$\Longleftrightarrow \begin{cases} a = A\cos\varphi & (1) \\ b = -A\sin\varphi & (2) \end{cases} \Longleftrightarrow \begin{cases} a^2+b^2 = A^2\cos^2\varphi + A^2\sin^2\varphi & (1)^2+(2)^2 \\ \dfrac{b}{a} = -\tan\varphi \end{cases}$$

$$\Longleftrightarrow \begin{cases} a^2+b^2 = A^2(\cos^2\varphi+\sin^2\varphi) = A^2 \\ \dfrac{b}{a} = -\tan\varphi \end{cases} \Longleftrightarrow \begin{cases} A = \sqrt{a^2+b^2} \\ \varphi = \arctan(-b/a) \end{cases}$$

3) • $f_1(t) = \cos t + \sin t$

$a = b = 1$. $A = \sqrt{2}$ et $\varphi = -\arctan(1) = -\pi/4$

Donc $f_1(t) = \sqrt{2}\cos(t-\pi/4)$

• $f_2(t) = \cos t - \sin t$

$a = 1$; $b = -1$; $A = \sqrt{2}$ et $\varphi = -\arctan(-1) = +\pi/4$

$f_2(t) = \sqrt{2}\cos(t+\pi/4)$

- $f_3(t) = \cos(\omega t) + \sqrt{3}\sin(\omega t)$

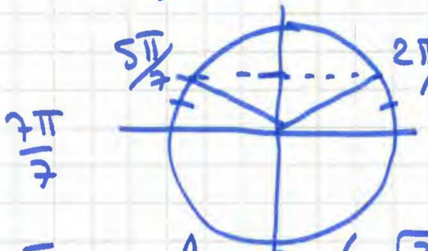
  $a = 1$ et $b = \sqrt{3}$ donc $A = 2$

  $\varphi = -\arctan(\sqrt{3}) = -\pi/3$

  Alors $f_3(t) = 2\cos(\omega t - \pi/3)$.

<u>Ex 10</u> $\text{Arcsin}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$; $\text{Arcsin}(0) = 0$;

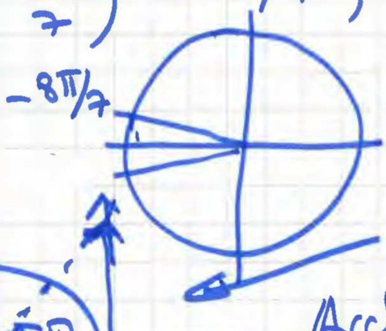
$\text{Arcsin}(\sin(-\pi/6)) = -\pi/6$; $\text{Arcsin}\left(\sin\frac{5\pi}{7}\right) = \frac{2\pi}{7}$



$\text{Arccos}(-1) = \pi$; $\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$; $\text{Arccos}(0) = \frac{\pi}{2}$

$\cos\left(\text{Arccos}\frac{113}{114}\right) = \frac{113}{114}$; $\text{Arccos}\left(\cos\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

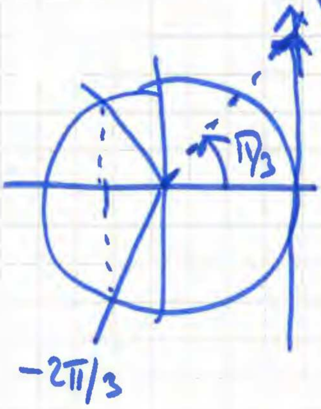
$\text{Arccos}\left(\cos\frac{-8\pi}{7}\right) = 6\pi/7$; $\text{Arctan}(\sqrt{3}) = \frac{\pi}{3}$; $\text{Arctan}(-1) = \frac{-\pi}{4}$



$\text{Arctan}(0) = 0$;

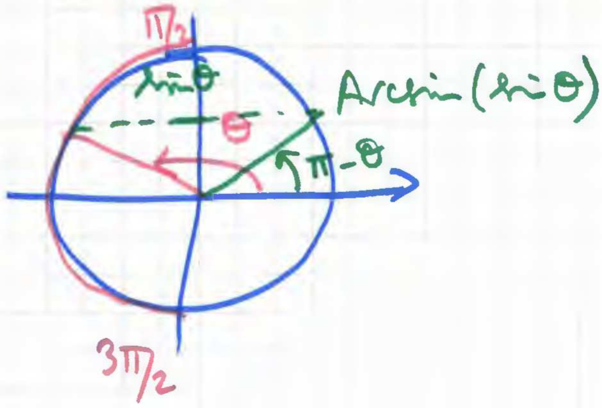
$\tan(\text{Arctan}\,189) = 189$

$\text{Arctan}(\tan(-2\pi/3)) = \pi/3$.



$\text{Arctan}\left(\tan\frac{\pi}{20}\right) = \frac{\pi}{20}$

$\forall\theta \in [\pi/2 ; 3\pi/2]$ $\text{Arcsin}(\sin\theta) = \pi - \theta$



Ex 11 $\sin(\operatorname{Arccos} x)$? $x \in [-1;1]$.

$\sin^2(\operatorname{Arccos} x) = 1 - \cos^2(\operatorname{Arccos} x)$

$\sin^2(\operatorname{Arccos} x) = 1 - x^2$

Alors $\sin(\operatorname{Arccos} x) = \pm\sqrt{1-x^2}$

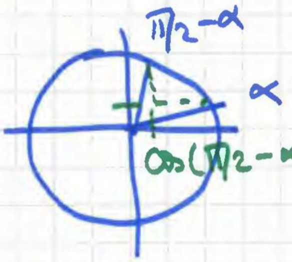
Comme $\operatorname{Arccos} x \in [0;\pi]$, alors $\sin(\operatorname{Arccos} x) \geq 0$

et $\sin(\operatorname{Arccos} x) = \sqrt{1-x^2}$. $\forall x \in [-1;1]$.

Ex 12 $\operatorname{Arccos} x + \operatorname{Arccos}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\iff \operatorname{Arccos} x = \frac{\pi}{2} - \operatorname{Arccos}\left(\frac{4}{5}\right)$

$\iff \cos(\operatorname{Arccos} x) = \cos\left(\pi/2 - \operatorname{Arccos}(4/5)\right)$



$\iff x = \sin(\operatorname{Arccos}(4/5))$

$\cos(\pi/2 - \alpha) = \sin\alpha$

$\iff x = \sqrt{1 - 16/25}$ d'après Ex 11.

$\iff x = \sqrt{9/25} = 3/5$.

Ex 13

- $\sin a = \sin\left(2\frac{a}{2}\right) = 2\sin(a/2)\cos(a/2)$ $\quad(\sin 2\theta = 2\sin\theta\cos\theta)$

$= 2\,\frac{\sin(a/2)}{\cos(a/2)}\cos^2(a/2)$ $\quad\left(1+\tan^2\theta = \frac{1}{\cos^2\theta}\right)$

$= 2\tan(a/2)\cdot\frac{1}{1+\tan^2(a/2)}$

$\sin a = \frac{2t}{1+t^2}$

- $\cos a = \cos\left(2\frac{a}{2}\right) = \cos^2(a/2) - \sin^2(a/2)$ $\quad\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \cos^2(a/2)\left(1 - \tan^2 a/2\right)$

$= \frac{1}{1+\tan^2(a/2)}\left(1-\tan^2 a/2\right)$

$$\cos a = \frac{1-t^2}{1+t^2}.$$

- $\tan a = \tan\left(2\,\frac{a}{2}\right) = \dfrac{\tan a/2 + \tan a/2}{1 - \tan a/2 \tan a/2} = \dfrac{2t}{1-t^2}$

Correction Ex. supplémentaires
Partie Complexes -

Ex7

a) $$z = \frac{1+j\tan\theta}{1-j\tan\theta} = \frac{1+j\frac{\sin\theta}{\cos\theta}}{1-j\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\cos\theta+j\sin\theta}{\cos\theta}}{\frac{\cos\theta-j\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta+j\sin\theta}{\cos\theta-j\sin\theta} = \frac{e^{j\theta}}{e^{-j\theta}}$$

$$z = e^{2j\theta} \quad \text{donc} \quad \begin{cases} |z| = 1 \\ \arg(z) = 2\theta. \end{cases}$$

b) $$z = e^{4jx} + e^{2jx} = e^{3jx}\left(e^{jx} + e^{-jx}\right)$$

$$z = \underbrace{2\cos x}_{>0 \text{ car } x\in[0;\pi]} . e^{3jx} \quad \text{donc} \quad \begin{cases} |z| = 2\cos x \\ \arg(z) = 3x. \end{cases}$$

c) $$z = \underbrace{\ln x}_{<0} . e^{jx} \qquad 0 < x < 1$$

$$= \underbrace{-\ln x}_{>0} . \left(-e^{jx}\right) = -\ln x . e^{j\pi} e^{jx}$$

$$z = -\ln x . e^{j(x+\pi)} \quad \text{donc} \quad \begin{cases} |z| = -\ln x \\ \arg(z) = x+\pi. \end{cases}$$

Ex8 $$C = \operatorname{Re}\left(1 + e^{j\theta} + e^{2j\theta} + \dots + e^{nj\theta}\right)$$

$$= \operatorname{Re}\left(\frac{1-(e^{j\theta})^{n+1}}{1-e^{j\theta}}\right)$$

$$\frac{1-e^{j(n+1)\theta}}{1-e^{jn\theta}} = \frac{e^{j\frac{n+1}{2}\theta}\left(e^{-j\frac{n+1}{2}\theta} - e^{j\frac{n+1}{2}\theta}\right)}{e^{j\frac{n}{2}\theta}\left(e^{-j\frac{n}{2}\theta} - e^{j\frac{n}{2}\theta}\right)}$$

$$= e^{j\frac{\theta}{2}} \frac{-2j\sin\left(\frac{n+1}{2}\theta\right)}{-2j\sin\left(\frac{n}{2}\theta\right)}$$

$$\dots = e^{j\frac{\theta}{2}} \frac{\sin\left(\frac{n+1}{2}\theta\right)}{\sin\left(\frac{n}{2}\theta\right)}$$

donc

$$C = \cos\theta/2 \times \frac{\sin\left(\frac{n+1}{2}\theta\right)}{\sin\left(\frac{n}{2}\theta\right)}.$$