Corrigé du DM7
1FTP

Exercice 1 :

* $f(x) = x^3 + 3x^2 + 3x - 1$ $\quad D_f = \mathbb{R}$

$f'(x) = 3x^2 + 6x + 3$ $\quad D_{f'} = \mathbb{R}$

* $f(x) = x + \frac{1}{x}$ $\quad D_f = \mathbb{R}^*$

$f'(x) = 1 - \frac{1}{x^2}$ $\quad D_{f'} = \mathbb{R}^*$

$\left(\frac{u}{v}\right)' = \frac{u'.v - uv'}{v^2}$

* $g(t) = \frac{t+1}{t-1}$ $\quad D_g = \mathbb{R} - \{1\}$

$g'(t) = \frac{t-1-t-1}{(t-1)^2} = \frac{-2}{(t-1)^2}$ $\quad D_{g'} = \mathbb{R} - \{1\}$

* $h(t) = \sqrt{t^2+1}$ $\quad D_h = \mathbb{R}$

$h'(t) = \dfrac{2t}{2\sqrt{t^2+1}} = \dfrac{t}{\sqrt{t^2+1}}$ $\quad D_{h'} = \mathbb{R}$

$(\sqrt{U})' = \dfrac{U'}{2\sqrt{U}}$

Méthode 1

$(\sqrt{U})' = \dfrac{U'}{2\sqrt{U}}$

* $I(w) = \dfrac{w}{\sqrt{w^2+1}}$ $\quad D_I = \mathbb{R}$

$I'(w) = \dfrac{u' \times v - v' \times u}{v^2}$ $\quad u = w \quad u' = 1 \quad v = \sqrt{w^2+1} \quad v' = \dfrac{2w}{2\sqrt{w^2+1}}$

$= \dfrac{\sqrt{w^2+1} - \dfrac{w}{\sqrt{w^2+1}} \times w}{\sqrt{w^2+1}^2}$

$$= \frac{\sqrt{w^2+1} - \frac{w^2}{\sqrt{w^2+1}}}{w^2+1} = \frac{\frac{w^2+1}{\sqrt{w^2+1}} - \frac{w^2}{\sqrt{w^2+1}}}{w^2+1}$$

$$= \frac{\frac{1}{\sqrt{w^2+1}}}{w^2+1} = \frac{1}{\sqrt{w^2+1}} \times \frac{1}{w^2+1}$$

$D_{I'} = \mathbb{R}$

Méthode 2

$$I(\omega) = \frac{\omega}{\sqrt{\omega^2+1}} = \omega(\omega^2+1)^{-\frac{1}{2}}$$

$$\begin{cases} (u.v)' = u'.v + uv' \\ (f^n)' = n\, f'\, f^{n-1} \end{cases}$$

$\mathcal{D}_I = \mathbb{R}$

$$I'(\omega) = \frac{1}{\sqrt{\omega^2+1}} - \frac{\omega^2}{(\omega^2+1)^{3/2}} = \frac{1}{\sqrt{\omega^2+1}\,(\omega^2+1)} = \frac{1}{(\omega^2+1)^{3/2}}$$

$$I'(\omega) = \omega'.(\omega^2+1)^{-1/2} + \omega.\left((\omega^2+1)^{-1/2}\right)'$$

$$= 1.(\omega^2+1)^{-\frac{1}{2}} + \omega.\frac{-1}{2}.2\omega.(\omega^2+1)^{-\frac{1}{2}-1}$$

$$= (\omega^2+1)^{-\frac{1}{2}} - \omega^2(\omega^2+1)^{-3/2}$$

$$I'(\omega) = \frac{1}{\sqrt{\omega^2+1}} - \frac{\omega^2}{\sqrt{\omega^2+1}\,(\omega^2+1)} = \frac{\omega^2+1-\omega^2}{\sqrt{\omega^2+1}\,(\omega^2+1)}$$

$\mathcal{D}_{I'} = \mathbb{R}$

$* \; X(\omega) = \Big(\underbrace{L\omega - \frac{1}{C\omega}}_{U}\Big)^2$ $\qquad (U^2)' = 2U'.U \qquad \mathcal{D}_X = \mathbb{R}^*.$

$U' = \left(L\omega - \frac{1}{C\omega}\right)' = L.(\omega)' - \frac{1}{C}.\left(\frac{1}{\omega}\right)' = L.1 - \frac{1}{C} \times -\frac{1}{\omega^2}$

constantes

$U' = L + \frac{1}{C\omega^2}$

donc $X'(\omega) = 2\left(L + \frac{1}{C\omega^2}\right)\left(L\omega - \frac{1}{C\omega}\right) \qquad \mathcal{D}_{X'} = \mathbb{R}^*$

$* \; Z(\omega) = \sqrt{R^2 + L^2\omega^2} \qquad D_Z = \mathbb{R}$

$(\sqrt{U})' = \frac{U'}{2\sqrt{U}}$

$Z'(\omega) = \frac{2L^2\omega}{2\sqrt{R^2+L^2\omega^2}} \qquad D_{Z'} = \mathbb{R}$

$\begin{cases} U' = (R^2 + L^2\omega^2)' \\ U' = 0 + L^2.2\omega \end{cases}$

$$* \quad i(t) = \underbrace{I \cdot \sqrt{2}}_{cte} \cdot \cos(\omega t + \varphi)$$

$$D_i = \mathbb{R}$$

$$i'(t) = -I \omega \sqrt{2} \cdot \sin(\omega t + \varphi)$$

$$D_{i'} = \mathbb{R}.$$

$$(\cos U)' = -U' \cdot \sin U$$

$$U' = (\omega t + \varphi)' = \omega + 0$$

$*$ $f_0(C) = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{L}} \times \dfrac{1}{\sqrt{C}} = \underbrace{\dfrac{1}{2\pi\sqrt{L}}}_{cte} \times C^{-1/2}$

$\mathcal{D}_{f_0} = \mathbb{R}^*_+$

$(x^n)' = n x^{n-1}$

$f_0'(C) = \dfrac{1}{2\pi\sqrt{L}} \cdot \dfrac{-1}{2} \cdot C^{-3/2}$

$C^{-3/2} = C^{-1} \times C^{-1/2}$

$= \dfrac{1}{C.\sqrt{C}}$

$= -\dfrac{1}{4\pi\sqrt{L}} \cdot \dfrac{1}{C\sqrt{C}}$

$f_0'(C) = -\dfrac{1}{4\pi C\sqrt{LC}}$ $\quad \mathcal{D}_{f_0'} = \mathbb{R}^*_+$

$*$ $W(C) = \dfrac{1}{2} \cdot \dfrac{Q^2}{C} = \underbrace{\dfrac{Q^2}{2}}_{cte} \cdot \dfrac{1}{C}$ $\quad \left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$ $\quad \mathcal{D}_W = \mathbb{R}^*$

$W'(C) = \dfrac{Q^2}{2} \times \dfrac{-1}{C^2} = -\dfrac{Q^2}{2C^2}$ $\quad \mathcal{D}_{W'} = \mathbb{R}^*$

$$\begin{cases} W(Q) = \dfrac{1}{2}\,\dfrac{Q^2}{C} = \underbrace{\dfrac{1}{2C}}_{cte} \cdot Q^2 \\[2ex] W'(Q) = \dfrac{Q}{C} \end{cases}$$

$\mathcal{D}_W = \mathcal{D}_{W'} = \mathbb{R}$.

$\mathcal{D}_V = \mathbb{R}$

$$\begin{cases} V(x) = \underbrace{\dfrac{\sigma}{2\varepsilon_0}}_{cte} \left(\sqrt{x^2+R^2} - x\right) \\[2ex] V'(x) = \dfrac{\sigma}{2\varepsilon_0} \left(\dfrac{x}{\sqrt{x^2+R^2}} - 1\right) \end{cases}$$

$\mathcal{D}_{V'} = \mathbb{R}$