DS2 - 2Appl. SUJET 1

Ex1 : $TZ(k.U(k)) = \frac{z}{(z-1)^2} = F(z)$

Donc $TZ(k^2.U(k)) = TZ(k.k.U(k)) = -zF'(z)$

$F'(z) = \frac{(z-1)^2 - 2z(z-1)}{(z-1)^4} = \frac{(z-1)(z-1-2z)}{(z-1)^4}$

$F'(z) = \frac{-z-1}{(z-1)^3}$ et $TZ(k^2.U(k)) = \frac{z^2+z}{(z-1)^3}$

* $TZ((k+2)^2.U(k-1)) = z^{-1}.TZ((k+3)^2)$

$F(z) = z^{-1}.(TZ(k^2) + 6TZ(k) + 9TZ(1))$

$= z^{-1}\left(\frac{z^2+z}{(z-1)^3} + 6.\frac{z}{(z-1)^2} + 9.\frac{z}{z-1}\right)$

$F(z) = \frac{z+1}{(z-1)^3} + \frac{6}{(z-1)^2} + \frac{9}{z-1}$

* $TZ\left(\frac{e^k}{e^{4k}}.U(k)\right) = TZ(e^{-k}) = TZ((e^{-1})^k)$

$= \frac{z}{z-\frac{1}{e}} = \frac{ez}{ez-1}$

Brouillon:

Ex2 $z^2 - 4z\sqrt{2} + 16 = z^2 - 2az\cos\omega + a^2$

ici $a = 4$; $2a\cos\omega = 4\sqrt{2} \iff \cos\omega = \frac{\sqrt{2}}{2}$

$\Leftarrow \omega = \frac{\pi}{4}$

De plus,

$z^2 - 2az\cos\omega = z^2 - 4z\frac{\sqrt{2}}{2} = z^2 - 2z\sqrt{2}$

Donc

$$T_z^{-1}\left(\frac{z - 2\sqrt{2}}{z^2 - 4z\sqrt{2} + 16}\right) = T_z^{-1}\left(z^{-1}\frac{z^2 - 2z\sqrt{2}}{z^2 - 4z\sqrt{2} + 16}\right)$$

$$f(k) = 4^{k-1}\cos\left((k-1)\frac{\pi}{4}\right)\cdot U(k-1)$$

Ex3 $Y(z) - z^{-1}Y(z) = 1$

$(1 - z^{-1})Y(z) = 1$

$$Y(z) = \frac{1}{1 - z^{-1}} \times \frac{z}{z}$$

$$Y(z) = \frac{z}{z - 1}$$

donc $y(k) = U(k)$ est la solution.

Exu :

1) $2Y(z) - 3z^{-1}Y(z) + z^{-2}Y(z) = X(z)$

$Y(z).(2 - 3z^{-1} + z^{-2}) = X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2 - 3z^{-1} + z^{-2}} \times \frac{z^2}{z^2}$$

$$H(z) = \frac{z^2}{2z^2 - 3z + 1.}$$

2) $x(k) = \delta(k) \Leftrightarrow X(z) = 1$

et $Y(z) = H(z).X(z) = \frac{z^2}{2z^2 - 3z + 1.}$

$Y(z) = z.G(z)$

où $G(z) = \frac{z}{2z^2 - 3z + 1}$

$\Delta = 9 - 8 = 1$

$z_1 = \frac{3+1}{4} = 1$ et $z_2 = \frac{3-1}{4} = \frac{1}{2}$

$$G(z) = \frac{z}{2(z-1)(z-1/2)} = \frac{z}{(z-1)(2z-1)}$$

$$G(z) = \frac{a}{z-1} + \frac{b}{2z-1}$$

$$a = \frac{1}{2-1} = 1 \text{ et } b = \frac{1/2}{1/2 - 1} = -1$$

Donc $G(z) = \frac{1}{z-1} - \frac{1}{2z-1}$

$Y(z) = \frac{z}{z-1} - \frac{z}{2(z-1/2)}$

Alors

la réponse impulsionnelle est:

$$y(k) = U(k) - \frac{1}{2} \cdot \left(\frac{1}{2}\right)^k \cdot U(k).$$

3) $x(k) = U(k-1) \Rightarrow X(z) = \frac{1}{z-1}$

$$Y(z) = H(z) \cdot X(z) = \frac{z}{(z-1)^2} - \frac{z}{2(z-1/2)(z-1)}$$

$$Y(z) = \frac{z}{(z-1)^2} + \frac{a}{z-1/2} + \frac{b}{z-1}$$

ici:

$$a = \frac{-1/2}{2(1/2-1)} = +1/2$$

$$b = \frac{-1}{2(1-1/2)} = -1.$$

Ainsi:

$$Y(z) = \frac{z}{(z-1)^2} + \frac{1/2}{z-1/2} - \frac{1}{z-1}$$

et $y(k) = k \cdot U(k) + \left(\frac{1}{2}\left(\frac{1}{2}\right)^{k-1} - 1\right) \cdot U(k-1)$