

DS 2 App - SUBJECT 2

$$\text{Ex 1} * f(k) = k^2 \cdot u(k)$$

$$F(z) = -z G'(z) \text{ ou } G(z) = Tz (k \cdot u(k))$$

$$F(z) = -z \cdot \frac{-z^{-1}}{(z-1)^3} = \frac{z^2 + z}{(z-1)^3}$$

$$* F(z) = z^{-1} \cdot Tz \left((k+1)^2 + 3(k+1) + 4 \right)$$

$$= z^{-1} \cdot Tz (k^2 + 5k + 8)$$

$$= z^{-1} \cdot \left(\frac{z^2 + z}{(z-1)^3} + 5 \frac{z}{(z-1)^2} + \frac{8z}{z-1} \right)$$

$$F(z) = \left(\frac{z+1}{(z-1)^3} + 5 \frac{1}{(z-1)^2} + \frac{8}{z-1} \right)$$

$$* F(z) = Tz \left(e^{ck} \cdot e \cdot u(k) \right) = e Tz \left((e^c)^k \right)$$

$$F(z) = e \frac{z}{z - e^2}$$

PROVILON

$$\text{Ex 2 } z^2 - 2z\sqrt{3} + 4 = z^2 - 2az \cos \omega + a^2$$

$$a^2 = 4 \Leftrightarrow a = 2$$

$$2az \cos \omega = 2z\sqrt{3} \Leftrightarrow \cos \omega = \frac{\sqrt{3}}{2} \Leftrightarrow \omega = \frac{\pi}{6}$$

$$z^2 - 2z\sqrt{3} = z^2 - \underbrace{2az \cos \omega}$$

$$\text{car } 2z \cdot \frac{\sqrt{3}}{2} = z\sqrt{3}$$

$$\text{Ainsi } \mathcal{T}_z \left(\frac{z^2 - 2z\sqrt{3}}{z^2 - 2z\sqrt{3} + 4} \right) = 2^k \cos \left(k \frac{\pi}{6} \right)$$

$$\text{Ex 3 } y(z) + z^{-1} y(z) = 3$$

$$(1 + z^{-1}) \cdot y(z) = 3$$

$$y(z) = \frac{3}{1 + z^{-1}} \times \frac{z}{z}$$

$$\text{Ainsi } y(z) = \frac{3z}{z+1}$$

$$\text{et } y(k) = 3 \cdot (-1)^k \cdot U(k)$$

Ex 4

$$\text{a) } y(z) - z^{-1} y(z) - 6z^{-2} y(z) = X(z)$$

$$(1 - z^{-1} - 6z^{-2}) y(z) = X(z)$$

Donc

$$H(z) = \frac{y(z)}{X(z)} = \frac{1}{1 - z^{-1} - 6z^{-2}} \times \frac{z^2}{z^2}$$

$$H(z) = \frac{z^2}{z^2 - z - 6}$$

$$b) x(k) = \delta(k) \Rightarrow X(z) = 1.$$

$$\text{Alors } Y(z) = H(z) \cdot X(z) = \frac{z^2}{z^2 - z - 6}$$

$$\Delta = 1 + 24 = 25.$$

$$\left\{ \begin{array}{l} z_1 = \frac{1+5}{2} = 3 \\ z_2 = \frac{1-5}{2} = -2 \end{array} \right.$$

$$Y(z) = \frac{z^2}{(z-3)(z+2)} = z \cdot G(z)$$

où

$$G(z) = \frac{z}{(z-3)(z+2)} = \frac{a}{z-3} + \frac{b}{z+2}$$

$$a = \frac{3}{3+2} = \frac{3}{5} \quad \text{et} \quad b = \frac{-2}{-2-3} = \frac{2}{5}.$$

Donc

$$Y(z) = \frac{3}{5} \frac{z}{z-3} + \frac{2}{5} \frac{z}{z+2}$$

et

$$y(k) = \left(\frac{3}{5} \cdot 3^k + \frac{2}{5} (-2)^k \right) U(k)$$

$$c) \mathcal{T}_z \left((-2)^k \cdot U(k) \right) = \frac{z}{z+2}$$

$$\text{et } \mathcal{T}_z \left(k \cdot (-2)^k \cdot U(k) \right) = -z \cdot \left(\frac{z}{z+2} \right)'$$

$$= -z \cdot \frac{z+2-z}{(z+2)^2} = -z \cdot \frac{2}{(z+2)^2}$$

$$d) x(k) = (-2)^{k-1} \cdot U(k-1) \Rightarrow X(z) = \frac{z^{-1}}{z+2}$$

$$\text{et } Y(z) = H(z) \cdot X(z) =$$

$$= \frac{3z}{5} \frac{1}{(z-3)(z+2)} + \frac{2}{5} \frac{z}{(z+2)^2}$$

$$= \frac{3z}{5} \left(\frac{a}{z-3} + \frac{b}{z+2} \right) + \frac{2}{5} \frac{z}{(z+2)^2}$$

$$= \frac{3z}{5} \left(+ \frac{1/5}{z-3} - \frac{1/5}{z+2} \right) + \frac{2}{5} \frac{z}{(z+2)^2}$$

$$Y(z) = \frac{3}{25} \left(\frac{z}{z-3} - \frac{z}{z+2} \right) - \frac{1}{5} \frac{-2z}{(z+2)^2}$$

Donc:

$$y(k) = \frac{3}{25} \left(3^k - (-2)^k \right) - \frac{1}{5} k (-2)^k$$

