Jocelyn
Helios
Mathis

TP3

Exercice 1 :

- $P = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \underbrace{\cos(5r)}_{\text{paire}} \, dr = \frac{2}{5}\int_0^{\frac{\pi}{6}} 5\cos(5r)\, dr$

$= \frac{2}{5}\left[\sin(5r)\right]_0^{\frac{\pi}{6}}$

$= \frac{2}{5}\left(\sin \frac{5\pi}{6} - \sin 0\right)$

$= \frac{2}{5}\left(\frac{1}{2} - 0\right) = \frac{1}{5}$

- $K = \int_0^{\pi} \sin(3r) \cdot \cos^8(3r)\, dr = -\frac{1}{3}\int_0^{\pi} -3\sin(3r) \cdot \cos^8(3r)\, dr$

$= -\frac{1}{3}\left[\frac{\cos^9(3r)}{9}\right]_0^{\frac{\pi}{3}}$

$= \frac{-\cos^9(\pi)}{27} + \frac{\cos^9(0)}{27}$

$= \frac{1}{27} + \frac{1}{27} = \frac{2}{27}$

- $P' = \int_0^{\frac{\pi}{3}} \sin\left(2r + \frac{\pi}{3}\right) dr = \frac{1}{2}\int_0^{\frac{\pi}{3}} 2\sin\left(2r + \frac{\pi}{3}\right) dr$

$= \frac{1}{2}\left[-\cos\left(2r + \frac{\pi}{3}\right)\right]_0^{\frac{\pi}{3}}$

$= \frac{1}{2} \cdot \left(-\cos\left(\frac{2\pi}{3} + \frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right)\right)$

$= \frac{1}{2} \cdot \left(-\cos \pi + \cos \frac{\pi}{3}\right)$

$= \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4}$

- $M(x) = \int \frac{x+2}{x^2 - 2x - 3}\, dx$

→ DSES $f(x) = \frac{x+2}{(x+1)(x-3)} = \frac{a}{x+1} + \frac{b}{x-3}$

$a = \left[(x+1) f(x)\right]_{x=-1} = \left[\frac{x+2}{x-3}\right]_{x=-1} = -\frac{1}{4}$

$b = \left[(x-3) f(x)\right]_{x=3} = \left[\frac{x+2}{x+1}\right]_{x=3} = \frac{5}{4}$

$f(x) = \frac{-\frac{1}{4}}{x+1} + \frac{\frac{5}{4}}{x-3}$

$M(x) = \int \left(-\frac{1}{4} \cdot \frac{1}{x+1} + \frac{5}{4} \cdot \frac{1}{x-3}\right) dx$

$= \int \left(-\frac{1}{4} \cdot \frac{1}{x+1}\right) dx + \int \left(\frac{5}{4} \cdot \frac{1}{x-3}\right) dx$

$= -\frac{1}{4} \times \ln(|x+1|) + \frac{5}{4} \cdot \ln(|x-3|) + cte$

$N(x) = \int \frac{x}{x^3 - x^2 + x - 1} dx$

$\rightarrow$ DSES : $f(x) = \frac{x}{(x-1)(x^2+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1}$

$a = [(x-1) f(x)]_{x=1} = \left[\frac{x}{x^2+1}\right]_{x=1} = \frac{1}{2}$

$bx + c = [(x^2+1) f(x)]_{x=j} = \left[\frac{x}{x-1}\right]_{x=j} = \frac{j}{j-1}$

$\frac{j}{-1+j} \times \frac{-j-1}{-1-j} = \frac{-j^2-j}{-1^2-j^2} = \frac{1-j}{2} = \frac{1}{2} - \frac{1}{2} j \qquad b = -\frac{1}{2} \; ; \; c = \frac{1}{2}$

$N(x) = \frac{1}{2} \int \frac{1}{x-1} dx - \frac{1}{2} \times \frac{1}{2} \int \frac{2x}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x^2+1} dx$

$= \frac{1}{2} \cdot \ln|x-1| - \frac{1}{4} \ln(x^2+1) + \frac{1}{2} \arctan x + cte$

$Q = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos x} dx = -1 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} -1 \frac{\sin x}{\cos x} dx$

$= -[\ln(|\cos x|)]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\ln(\cos \frac{\pi}{3}) + \ln(\cos \frac{\pi}{6})$

$= -\ln\left(\frac{1}{2}\right) + \ln\left(\frac{\sqrt{3}}{2}\right)$

$= \ln\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \ln(\sqrt{3})$

Exercice 2 :

$K = \int_0^{\frac{\pi}{2}} e^{-x} \cos x \, dx$

$u = e^{-x} \qquad u' = -e^{-x}$

$v' = \cos x \qquad v = \sin x$

$\int u v' = [u v] - \int u' v$

$$K = \int_0^{\frac{\pi}{2}} e^{-x} \cos x \, dx = \left[e^{-x} \cdot \sin x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} - e^{-x} \sin x \; dx$$

$$= \left(e^{-\frac{\pi}{2}} \cdot \sin \frac{\pi}{2} \right) - \left\{ \left[e^{-x} \cdot \cos x \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} e^{-x} \cdot \cos x \, dx \right\}$$

$$\begin{aligned} u &= -e^{-x} & u' &= e^{-x} \\ v' &= \sin x & v &= -\cos x \end{aligned}$$

$$= e^{-\frac{\pi}{2}} + 1 - \underbrace{\int_0^{\frac{\pi}{2}} e^{-x} \cos \, dx}_{K}$$

$$K = e^{-\frac{\pi}{2}} + 1 - K \iff 2K = e^{-\frac{\pi}{2}} + 1 \iff K = \frac{e^{-\frac{\pi}{2}} + 1}{2}$$