Fernandez Helios
Benjamin Rocheblaine

TP n°4

groupe C

Exercice 2 page 19 :

$J = \int_0^1 \arctan(x)\,dx$ $\quad \begin{cases} u = \arctan(x) \\ v' = 1 \end{cases} \Rightarrow \begin{cases} u' = \frac{1}{1+x^2} \\ v = x \end{cases}$

Ainsi par IPP : $J = \left[\arctan(x) . x\right]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$

$J = \arctan(1) - \arctan(0) - \left[\frac{\ln|1+x^2|}{2}\right]_0^1$

$J = \frac{\pi}{4} - \ln(2)$

$K = \int_0^{\pi/2} e^{-x} . \cos(x)\,dx$ $\quad \begin{cases} u = e^{-x} \\ v' = \cos(x) \end{cases} \Rightarrow \begin{cases} u' = -e^{-x} \\ v = \sin(x) \end{cases}$

Par IPP : $K = \left[e^{-x} \sin x\right]_0^{\pi/2} - \int_0^{\pi/2} -e^{-x} \sin x\,dx$

$K = e^{-\pi/2} - 0 + \int_0^{\pi/2} e^{-x} \sin x\,dx$

2ème IPP : $\begin{cases} u = e^{-x} \\ v' = \sin(x) \end{cases} \Rightarrow \begin{cases} u' = -e^{-x} \\ v = -\cos x \end{cases}$

Ainsi : $K = e^{-\pi/2} + \left[-\cos x . e^{-x}\right]_0^{\pi/2} - \int_0^{\pi/2} e^{-x} . \cos x\,dx$

$K = e^{-\pi/2} + 1 - K$

$2K = e^{-\pi/2} + 1$

$K = \frac{e^{-\pi/2} + 1}{2}$

$L(x) = \int \frac{x^2}{(x^2+1)^{3/2}}\,dx$

$= \int x^2 \times (x^2+1)^{-3/2}\,dx = \int \frac{x}{2} \times 2x\,(x^2+1)^{-3/2}\,dx$

Résolution par IPP : $\begin{cases} U(x) = \frac{x}{2} \\ V'(x) = 2x\,(x^2+1)^{-3/2} \end{cases}$ $\quad \begin{cases} U'(x) = \frac{1}{2} \\ V(x) = \frac{(x^2+1)^{-1/2}}{-1/2} = -2\,(x^2+1)^{-1/2} \end{cases}$

$L = (x) = \int \frac{x^2}{(x^2+1)^{3/2}}\,dx = -\left[x \times (x^2+1)^{-1/2}\right] + \int (x^2+1)^{-1/2}\,dx$

$L(x) = -x \times (x^2+1)^{-1/2} + \operatorname{argsh}(x) + cte$

Exercice 4 :

$M(x) = \int x^2 . e^{3x}\,dx$ $\quad \begin{cases} U(x) = x^2 \\ V'(x) = e^{3x} \end{cases}$ $\quad \begin{cases} U'(x) = 2x \\ V(x) = \frac{1}{3} e^{3x} \end{cases}$

PAR IPP :

$M(x) = \int x^2 . e^{3x}\,dx = \frac{1}{3}\left[x^2 . e^{3x}\right] - \frac{2}{3}\int x . e^{3x}\,dx$

$\int x . e^{3x}\,dx \rightarrow \begin{cases} U(x) = x \\ V'(x) = e^{3x} \end{cases}$ $\quad \begin{cases} U'(x) = 1 \\ V(x) = \frac{1}{3} e^{3x} \end{cases}$

$$\int x \cdot e^{3x}\,dx = \frac{x \cdot e^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx = \frac{x \cdot e^{3x}}{3} - \frac{1}{9}e^{3x}$$

$$\Rightarrow M(x) = \int x^2 \cdot e^{3x}\,dx = \frac{x^2 \times e^{3x}}{3} - \frac{2}{3}\left(\frac{x \times e^{3x}}{3} - \frac{1}{9}e^{3x}\right)$$

$$M(x) = \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{9}x \cdot e^{3x} + \frac{2}{27}e^{3x} = e^{3x}\left(\frac{x^2}{3} - \frac{2}{9}x + \frac{2}{27}\right) + \text{cte}$$

$$V(t) = \int (t^3 + 2t + 3) \cdot \ln|t|\,dt \qquad \begin{cases} u = \ln|t| \\ v' = t^3 + 2t + 3 \end{cases} \Rightarrow \begin{cases} u' = \frac{1}{t} \\ v = \frac{t^4}{4} + t^2 + 3t \end{cases}$$

$$V(t) = \ln|t| \cdot \left(\frac{t^4}{4} + t^2 + 3t\right) - \int \frac{t^3}{4} + t + 3\,dt$$

$$= \ln|t| \cdot \left(\frac{t^4}{4} + t^2 + 3t\right) - \frac{t^4}{16} - \frac{t^2}{2} - 3t + \text{cte}$$

$$K(x) = \int (1 + \tan^2 x) \cdot \tan^3 x\,dx = \frac{\tan^4 x}{4} + \text{cte}$$

$$L(x) = \int \frac{1}{x^2} e^{1/x} = -e^{1/x} + \text{cte}$$

$$N = \int_0^1 \frac{1}{e^{-t}+1}\,dt = \int_0^1 \frac{e^t}{1+e^t}\,dt = \left[\ln|e^t+1|\right]_0^1 = \ln|e+1| - \ln(2) = \ln\left(\frac{e+1}{2}\right)$$