Révisions DS1 du semestre 2

Quelle est la formule juste ?

1. $\sin(pt) = \frac{e^{jpt} + e^{-jpt}}{2}$ — 1%
2. $\sin(pt) = \frac{e^{jpt} - e^{-jpt}}{2}$ — 2%
3. $\sin(pt) = \frac{e^{-jpt} - e^{-jpt}}{2j}$ — 3%
4. $\sin(pt) = \frac{e^{jpt} \times e^{-jpt}}{2j}$ — 4%
5. $\sin(pt) = \frac{e^{jpt} - e^{-jpt}}{2j}$ — 5%

Quelle est la formule juste ?

1. $\sin(3t).\sin(4t) = \dfrac{e^{jt}+e^{-jt}-e^{7jt}-e^{-7jt}}{4}$ — 1%
2. $\sin(3t).\sin(4t) = \dfrac{-e^{jt}-e^{-jt}+e^{7jt}+e^{-7jt}}{2j}$ — 2%
3. $\sin(3t).\sin(4t) = \dfrac{e^{7jt}-e^{-jt}-e^{jt}-e^{-7jt}}{-4}$ — 3%
4. *Aucune des réponses n'est juste* — 4%

$$\sin(3t).\sin(4t) = \frac{e^{3jt} - e^{-3jt}}{2j} \times \frac{e^{4jt} - e^{-4jt}}{2j}$$

$$= -\frac{1}{4}\left(e^{7jt} - e^{-jt} - e^{jt} + e^{-7jt}\right)$$

$$= \frac{1}{4}\left(-e^{7jt} + e^{-jt} + e^{jt} - e^{-7jt}\right)$$

$$\sin(3t).\sin(4t) = \frac{1}{4}\left(e^{jt} + e^{-jt} - e^{7jt} - e^{-7jt}\right)$$

Quelle est la linéarisation de sin(3t).sin(4t) ?

1. $\sin(3t).\sin(4t) = \frac{\cos(t) - \cos(7t)}{2}$ — 1%
2. $\sin(3t).\sin(4t) = \frac{\cos(t) - \cos(7t)}{4}$ — 2%
3. $\sin(3t).\sin(4t) = \frac{\cos(t) + \sin(7t)}{4}$ — 3%
4. *Aucune des réponses n'est juste* — 4%

$$\sin(3t)\cdot\sin(4t) = \frac{1}{4}\left(e^{jt} + e^{-jt} - e^{7jt} - e^{-7jt}\right)$$

$$= \frac{1}{4}\left[e^{jt} + e^{-jt} - \left(e^{7jt} + e^{-7jt}\right)\right]$$

$$= \frac{1}{4}\left(2\cos t - 2\cos(7t)\right) = \frac{1}{4}\times 2\left(\cos t - \cos(7t)\right)$$

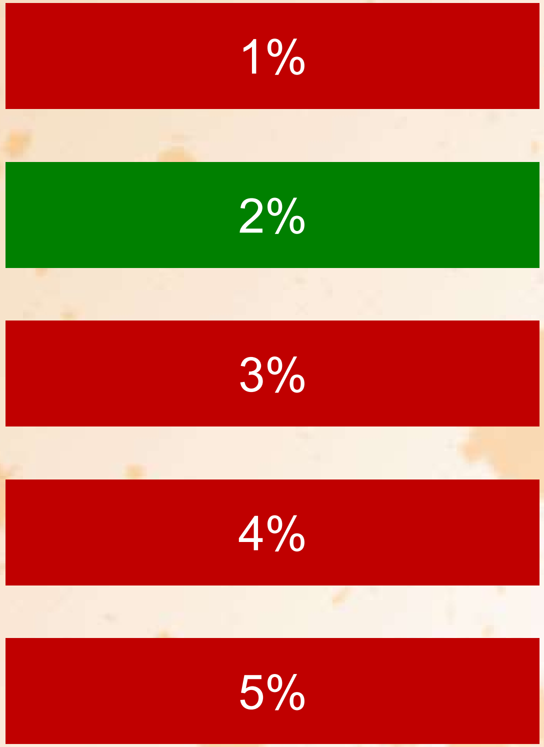
$$\sin(3t)\cdot\sin(4t) = \frac{1}{2}\left(\cos t - \cos(7t)\right)$$

Q

Quelle est la valeur de

$$I = \int_{-\pi/2}^{\pi/2} \sin(3t)\,.\,\sin(4t)\,dt\ ?$$

1. $I = 0$
2. ✓2 $I = \frac{8}{7}$
3. $I = -\frac{8}{7}$
4. $I = 8$
5. *Aucune réponse n'est juste*

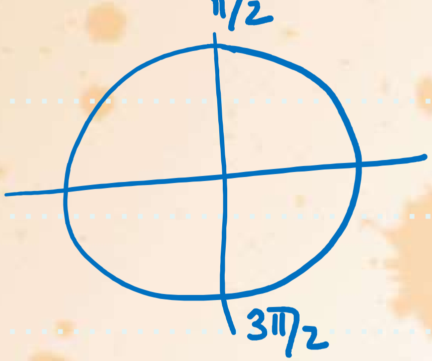


$$I = \int_{-\pi/2}^{\pi/2} \underbrace{\sin(3t)}\cdot\underbrace{\sin(4t)}\,dt = 2\times\int_{0}^{\pi/2}\sin(3t)\cdot\sin(4t)\,dt$$

impaire x impaire = paire

$$I = \cancel{2}\int_{0}^{\pi/2}\frac{\cos t-\cos(7t)}{\cancel{2}}\,dt$$

$$I = \left[\sin t-\frac{\sin(7t)}{7}\right]_{0}^{\pi/2}$$



$$= \sin\left(\frac{\pi}{2}\right)-\frac{\sin(7\pi/2)}{7}-\left(\sin 0-\frac{\sin 0}{7}\right)$$

$$I = 1+\frac{1}{7}=\frac{8}{7}$$

Pourquoi l'intégrale

$$J = \int_0^{2\pi} sin^3(t)dt \quad vaut\ 0\ ?$$

1. $car\ \sin(0) = \sin(2\pi) = 0$ — 1%
2. $car\ \cos(0) = \cos(2\pi) = 1$ — 2%
3. $car\ \sin\ est\ impaire$ — 3%
4. ✓ $Aucune\ des\ justifications\ n'est\ suffisante$ — 4%

$$J = \int_0^{2\pi} \underbrace{\sin^3(t)}_{2\pi \text{ périodique}} \, dt = \int_{-\pi}^{\pi} \underbrace{\sin^3(t)}_{\text{impaire}} \, dt = 0$$

Quelle est la vraie formule d'IPP ?

1. $[UV]_a^b - \int_a^b U'V dt$ — 1%
2. $[U'V']_a^b - \int_a^b U'V dt$ — 2%
3. $[UV]_a^b - \int_a^b UV' dt$ — 3%
4. $Aucune\ réponse\ n'est\ la\ formule\ d'IPP$ — 4%

$$\int_a^b u.v'\,dt = [u.v]_a^b - \int_a^b u'v\,dt$$

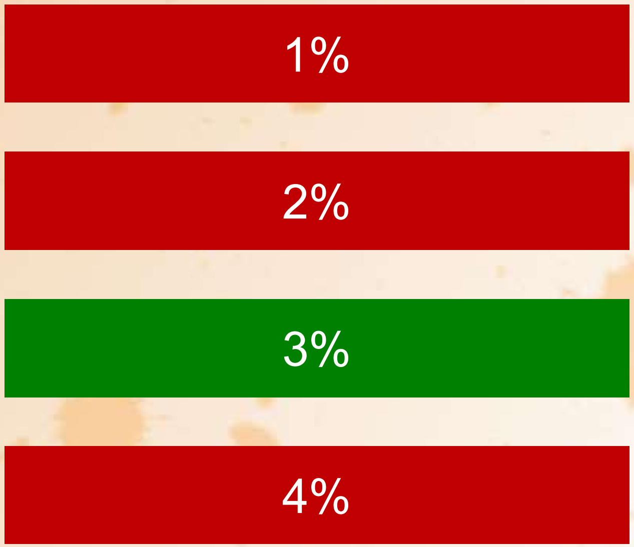
ou

$$\int_a^b u'.v\,dt = [u.v]_a^b - \int_a^b u.v'\,dt$$

Pour calculer par IPP

$$K = \int_0^1 (3t-5).e^{-2t}\,dt \text{ on pose :}$$

1. $U = e^{-2t}$ et $V' = 3t - 5$
2. $V = e^{-2t}$ et $U' = 3t - 5$
3. $U = 3t - 5$ et $V' = e^{-2t}$
4. *Tout est faux*



$$K = \int_0^1 \underbrace{(3t-5)}_{P} \underbrace{e^{-2t}}_{E} \, dt$$

ALPES

$$\begin{cases} U = 3t-5 \\ V' = e^{-2t} \end{cases}$$

Rappel :

$$\xrightarrow{\text{choix de } U}$$

ALPES

Arctan, Ln, poly., Exp., Cos., sin.

$$\int_a^b U.V' \, dt = [U.V]_a^b - \int_a^b U'.V \, dt$$

Pour calculer par IPP

$$K = \int_0^1 (3t-5).e^{-2t}\,dt \ \ on\ obtient :$$

1. $K = -2[(3t-5)e^{-2t}]_0^1 + 6\int_0^1 e^{-2t}dt$ — 1%
2. $K = -\frac{1}{2}[(3t-5)e^{-2t}]_0^1 + 6\int_0^1 e^{-2t}dt$ — 2%
3. $K = -\frac{1}{2}[(3t-5)e^{-2t}]_0^1 + \frac{3}{2}\int_0^1 e^{-2t}dt$ — 3%
4. $Tout\ est\ faux$ — 4%

$$K = \int_0^1 \underbrace{(3t-5)}_{P}\,\underbrace{e^{-2t}}_{E}\,dt$$

ALPES

$$\begin{cases} U = 3t-5 \\ V' = e^{-2t} \end{cases} \Rightarrow \begin{cases} U' = 3 \\ V = \dfrac{e^{-2t}}{-2} \end{cases}$$

$$K = [U.V]_a^b - \int_a^b U'V\,dt = -\frac{1}{2}\left[(3t-5)e^{-2t}\right]_0^1 + \frac{3}{2}\int_0^1 e^{-2t}\,dt$$

La valeur de $K = \int_0^1 (3t-5).e^{-2t}\,dt$ *est donc* :

1. $K = \frac{1}{4}(e^{-2}+7)$ — 1%
2. $K = \frac{1}{4}(-e^{-2}-7)$ — 2%
3. $K = \frac{1}{4}(e^{-2}-7)$ — 3%
4. *Aucune des valeurs n'est exacte* — 4%

$$K = \int_0^1 \underbrace{(3t-5)}_{P}\ \underbrace{e^{-2t}}_{E}\, dt$$

ALPES

$$\begin{cases} U = 3t-5 \\ V' = e^{-2t} \end{cases} \Rightarrow \begin{cases} U' = 3 \\ V = \dfrac{e^{-2t}}{-2} \end{cases}$$

$$K = [U.V]_a^b - \int_a^b U'V\,dt = -\frac{1}{2}\left[(3t-5)e^{-2t}\right]_0^1 + \frac{3}{2}\int_0^1 e^{-2t}\,dt$$

$$K = -\frac{1}{2}\left(-2e^{-2} + 5\right) + \frac{3}{2}\left[\frac{e^{-2t}}{-2}\right]_0^1$$

$$= e^{-2} - \frac{5}{2} - \frac{3}{4}\left(e^{-2} - 1\right)$$

$$K = e^{-2} - \frac{3}{4}e^{-2} - \frac{5}{2} + \frac{3}{4} = \frac{e^{-2}}{4} - \frac{7}{4} = \frac{1}{4}\left(e^{-2} - 7\right)$$

La DSES de $f(x) = \frac{x}{x^2-2x-15}$

$est\ de\ la\ forme\ :$

1. $f(x) = \frac{a}{x+3} + \frac{b}{x-5}$ — 1%
2. $f(x) = \frac{a}{x-3} + \frac{b}{x+5}$ — 2%
3. $f(x) = \frac{a}{x^2} + \frac{b}{2x} + \frac{c}{15}$ — 3%

$$f(x) = \frac{x}{x^2-2x-15} = \frac{A(x)}{B(x)}$$

→ Factorisation de B : $\Delta = (-2)^2 - 4(1)(-15) = 4 + 60 = 64$

$$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{2+8}{2} = \frac{10}{2} = 5 \quad \text{et} \quad x_2 = \frac{-b-\sqrt{\Delta}}{2a} = -3$$

OU

$$x^2 - Sx + P = x^2 - 2x - 15.$$

$$\begin{cases} x_1 + x_2 = 2 \\ x_1 \times x_2 = -15. \end{cases}$$

$$f(x) = \frac{x}{(x+3)(x-5)} = \frac{a}{x+3} + \frac{b}{x-5}$$

Les primitives de $f(x) = \frac{x}{x^2-2x-15}$ $sont$:

1. $F(x) = \frac{1}{8}(3ln|x+3| + 5ln|x-5|) +$ 1%
2. $F(x) = \frac{1}{8}(5ln|x+3| + 3ln|x-5|) +$ 2%
3. $F(x) = \frac{1}{8}(-3ln|x+3| + 5ln|x-5|)$ 3%
4. $Aucune\ des\ réponses\ n'est\ juste$ 4%

$$f(x) = \frac{x}{(x+3)(x-5)} = \frac{a}{x+3} + \frac{b}{x-5}$$

$$a = \left[(x+3)f(x)\right]_{x=-3} = \left[\frac{x}{x-5}\right]_{x=-3} = \frac{-3}{-8} = \frac{3}{8}$$

$$b = \left[(x-5)f(x)\right]_{x=5} = \left[\frac{x}{x+3}\right]_{x=5} = \frac{5}{8}$$

$$f(x) = \frac{3}{8} \cdot \frac{1}{x+3} + \frac{5}{8} \frac{1}{x-5}.$$

$$\int f(x)\,dx = \frac{3}{8} \cdot \ln|x+3| + \frac{5}{8} \cdot \ln|x-5| + cte$$

$$= \frac{1}{8}\left(3 \cdot \ln|x+3| + 5 \cdot \ln|x-5|\right) + cte$$

La DSES de $f(x) = \dfrac{x+3}{(x-1)^3(x+2)}$ *est de la forme :*

1. $f(x) = \dfrac{a}{x-1} + \dfrac{b}{x-1} + \dfrac{c}{x-1} + \dfrac{d}{x+2}$ — 1%
2. $f(x) = \dfrac{a}{(x-1)^3} + \dfrac{b}{(x-1)^2} + \dfrac{c}{x-1} + \dfrac{d}{x+2}$ — 2%
3. $f(x) = \dfrac{a}{(x-1)^3} + \dfrac{b}{x+2}$ — 3%
4. *Je n'en sais rien* — 4%

Q

$$f(x) = \frac{x+3}{(x-1)^3(x+2)} = \frac{a}{(x-1)^3} + \frac{b}{(x-1)^2} + \frac{c}{x-1} + \frac{d}{x+2}$$

✓1

1. $a = \frac{4}{3}$ $et\ d = \frac{-1}{27}$ — 1%
2. $a = \frac{-1}{27}$ $et\ d = \frac{4}{3}$ — 2%
3. $a = \frac{4}{3}$ $et\ d = \frac{1}{27}$ — 3%
4. $a = \frac{1}{27}$ $et\ d = \frac{4}{3}$ — 4%

$$f(x) = \frac{x+3}{(x-1)^3(x+2)} = \frac{a}{(x-1)^3} + \frac{b}{(x-1)^2} + \frac{c}{x-1} + \frac{d}{x+2}$$

$$a = \left[(x-1)^3 . f(x)\right]_{x=1} = \left[\frac{x+3}{x+2}\right]_{x=1} = \frac{4}{3}$$

$$d = \left[(x+2)\, f(x)\right]_{x=-2} = \left[\frac{x+3}{(x-1)^3}\right]_{x=-2} = \frac{1}{(-3)^3} = -\frac{1}{27}$$

Et pour b et c ? On ne peut pas appliquer ces formules car :

$$b = \left[(x-1)^2 . f(x)\right]_{x=1} = \left[\frac{x+3}{(x-1)(x+2)}\right]_{x=1}$$ est impossible : division par 0

Donc pour c :

$$c = \left[(x-1)\, f(x)\right]_{x=1} = \left[\frac{x+3}{(x-1)^2(x+2)}\right]_{x=1} =$$

Q

$$\frac{x+3}{(x-1)^3(x+2)} = \frac{4}{3(x-1)^3} + \frac{b}{(x-1)^2} + \frac{c}{x-1} + \frac{-1}{27(x+2)}$$

1. $b = \frac{1}{9}$ et $c = -\frac{1}{27}$ — 1%
2. $b = \frac{1}{9}$ et $c = \frac{1}{27}$ — 2%
3. $b = -\frac{1}{27}$ et $c = \frac{1}{9}$ — 3%
4. $b = -\frac{1}{9}$ et $c = -\frac{1}{27}$ — 4%

$$f(x)=\frac{x+3}{(x-1)^3(x+2)}=\frac{4}{3(x-1)^3}+\frac{b}{(x-1)^2}+\frac{c}{x-1}-\frac{1}{27(x+2)}$$

Calcul de b et c, par exemple en posant $x=0$: 1ère équation

$$f(0)=\frac{3}{-2}=\frac{4}{-3}+\frac{b}{1}+\frac{c}{-1}-\frac{1}{54} \iff b-c=-\frac{3}{2}+\frac{4}{3}+\frac{1}{54}$$

$$\iff b-c=-\frac{4}{27} \quad (1)$$

2ème équation :

$$\lim_{x\to+\infty} x f(x)=\lim_{x\to+\infty}\frac{x^2}{x^4}=\lim_{x\to+\infty}\frac{4x}{3x^3}+\frac{bx}{x^2}+\frac{cx}{x}-\frac{x}{27x}$$

$$0=0+0+c-\frac{1}{27} \iff c=\frac{1}{27} \quad (2)$$

et d'après (1) $b=-\frac{4}{27}+\frac{1}{27}=-\frac{3}{27}=-\frac{1}{9}$

$$f(x)=\frac{4}{3(x-1)^3}-\frac{1}{9(x-1)^2}+\frac{1}{27(x-1)}-\frac{1}{27(x+2)} \quad \forall x\neq 1;-2$$

Quelles sont les primitives de $\dfrac{1}{(x-1)^3}$?

1. $\ln\left((x-1)^3\right) + Cte$
2. $\arctan(x) + Cte$
3. ✓3 *Aucune réponse n'est juste*
4. $-\dfrac{1}{x-1} + Cte$

1%

2%

3%

4%

$$\int \frac{1}{(x-1)^3}\,dx = \ ?$$

Formule : $$\int U'U^{\alpha}\,dx = \frac{U^{\alpha+1}}{\alpha+1} + cte \quad \alpha \neq -1.$$

Ici $$\int \frac{1}{(x-1)^3}\,dx = \int (x-1)^{-3}\,dx.$$

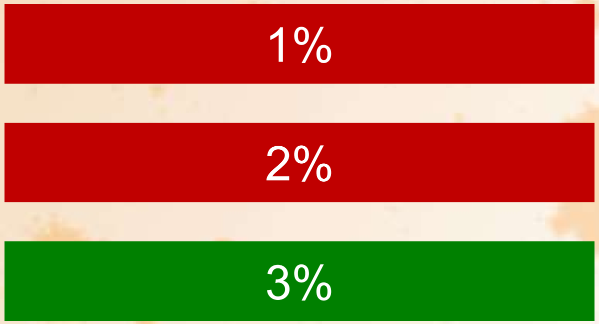
$\alpha = -3$; $U = x-1 \Longrightarrow U' = 1.$

donc $$\int \frac{1}{(x-1)^3} = \frac{(x-1)^{-2}}{-2} + cte = -\frac{1}{2(x-1)^2} + cte$$

On souhaite calculer l'intégrale I = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3t)e^{-t}dt$

Peut-on écrire que :

1. I = $2.\int_{0}^{\frac{\pi}{3}} \cos(3t)e^{-t}dt$?
2. $I = 0$?
3. *Ni la réponse* 1 *ni la* 2 *n'est juste*.



$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \underbrace{\cos(3t)}_{\text{pair}}\, \underbrace{e^{-t}}_{\text{ni pair, ni impair}}\, dt$$

centré en 0

Notes

Ni 1) Ni 2)

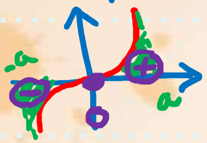
Si $f(t) = \cos(3t) \cdot e^{-t}$ alors $f(-t) = \cos(-3t) \cdot e^{t} = \cos(3t)\, e^{t}$

Contre Exple : $f(-\pi) = \cos(3\pi) e^{\pi} = -e^{\pi}$

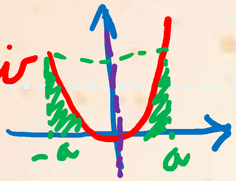
$f(\pi) = \cos(-3\pi) = e^{-\pi} = -e^{-\pi} = -\frac{1}{e^{\pi}} \neq f(-\pi)$ et $\neq -f(-\pi)$

f est donc ni pair, ni impair.

Rappel : $\int_{-a}^{a} f(x)\,dx = 0$ si f est impair



$\int_{-a}^{a} f(x)\,dx = 2 \times \int_{0}^{a} f(x)\,dx$ si f est pair



Pour calculer l'intégrale I = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3t)e^{-t}dt$ à l'aide d'une IPP,

on pose $\mathrm{U} = e^{-t}$ et $\mathrm{V}' = \cos(3t)$ et on obtient :

1. $I = \frac{-1}{3}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin(3t)e^{-t}dt$ — 1%
2. $I = 3\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin(3t)e^{-t}dt$ — 2%
3. ✓3 $I = \frac{1}{3}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin(3t)e^{-t}dt$ — 3%
4. $I = -3\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin(3t)e^{-t}dt$ — 4%

Pour calculer l'intégrale I = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3t)e^{-t}dt$ à l'aide d'une IPP,

on pose $\mathrm{U} = e^{-t}$ et $\mathrm{V}' = \cos(3t)$ et on obtient :

Notes

$$U' = -e^{-t} \qquad V = \frac{\sin(3t)}{3}$$

$$I = [U.V]_a^b - \int_a^b U'.V\,dt$$

$$I = \frac{1}{3}\left[e^{-t}.\sin(3t)\right]_{-\pi/3}^{\pi/3} + \frac{1}{3}\int_{-\pi/3}^{\pi/3} \sin(3t).e^{-t}\,dt$$

$$I = \underbrace{\frac{1}{3}\left(e^{-\pi/3}.\sin(\pi) - e^{\pi/3}.\sin(-\pi)\right)}_{0} + \frac{1}{3}.\int_{-\pi/3}^{\pi/3} \sin(3t)e^{-t}\,dt$$

Pour calculer l'intégrale I = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3t)e^{-t}dt = \frac{1}{3}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin(3t)e^{-t}dt$

à l'aide d'une deuxième IPP, on obtient :

1. $I = \frac{1}{9}\left\{e^{-\frac{\pi}{3}} - e^{\frac{\pi}{3}} - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} cos(3t)e^{-t}dt\right\}$ — 1%
2. $I = \frac{1}{9}\left\{e^{-\frac{\pi}{3}} - e^{\frac{\pi}{3}} + \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} cos(3t)e^{-t}dt\right\}$ — 2%
3. $I = \frac{1}{9}\left\{e^{\frac{\pi}{3}} - e^{-\frac{\pi}{3}} - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} cos(3t)e^{-t}dt\right\}$ — 3%
4. $I = \frac{1}{9}\left\{e^{\frac{\pi}{3}} - e^{-\frac{\pi}{3}} + \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} cos(3t)e^{-t}dt\right\}$ — 4%

Pour calculer l'intégrale I = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3t) e^{-t} dt = \frac{1}{3}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin(3t) e^{-t} dt$

à l'aide d'une deuxième IPP, on obtient :

Notes

On pose $u = e^{-t}$ $\quad u' = -e^{-t}$

$v' = \sin(3t)$ $\quad v = -\frac{\cos(3t)}{3}$

$$I = [u.v]_a^b - \int_a^b u'v\,dt$$

$$I = \frac{1}{3}\left\{ -\frac{1}{3}\left[e^{-t}.\cos(3t)\right]_{-\pi/3}^{\pi/3} - \frac{1}{3}\int_{-\pi/3}^{\pi/3} \cos(3t).e^{-t}\,dt \right\}$$

$$= \frac{1}{3}\left\{ -\frac{1}{3}\left(e^{-\pi/3}\underbrace{\cos\pi}_{-1} - e^{\pi/3}\underbrace{\cos(-\pi)}_{-1}\right) - \frac{1}{3}\int^{\pi/3} \cos(3t)e^{-t}\,dt \right\}$$

$$I = \frac{1}{3}\left\{ \frac{1}{3}\left(e^{-\pi/3} - e^{-\pi/3}\right) - \frac{1}{3} I \right\} = \frac{1}{9}\left\{ e^{-\pi/3} - e^{\pi/3} - I \right\}$$

Après deux IPP on a alors :

$$\mathrm{I} = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3t)e^{-t}dt = \frac{1}{9}\left\{e^{-\frac{\pi}{3}} - e^{\frac{\pi}{3}} - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} cos(3t)e^{-t}dt\right\} \text{ donc :}$$

1. $I = \frac{-1}{10}$ — 1%
2. $I = \frac{1}{9}\left(e^{\frac{-\pi}{3}} - e^{\frac{\pi}{3}}\right)$ — 2%
3. $I = \frac{1}{8}\left(e^{\frac{-\pi}{3}} - e^{\frac{\pi}{3}}\right)$ — 3%
4. $I = \frac{1}{10}\left(e^{\frac{-\pi}{3}} - e^{\frac{\pi}{3}}\right)$ — 4%
5. $I = \frac{1}{10}$ — 5%

✓4

Après deux IPP on a alors :

I = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3t)e^{-t}dt = \frac{1}{9}\left\{e^{-\frac{\pi}{3}} - e^{\frac{\pi}{3}} - \underbrace{\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} cos(3t)e^{-t}dt}_{I}\right\}$ donc :

Notes

$$I = \frac{1}{9}\left(e^{-\pi/3} - e^{\pi/3}\right) - \frac{1}{9} I$$

$$I\underbrace{\left(1+\frac{1}{9}\right)}_{\frac{10}{9}} = \frac{1}{9}\left(e^{-\pi/3} - e^{\pi/3}\right)$$

$$I = \frac{9}{10} \times \frac{1}{9}\left(e^{-\pi/3} - e^{\pi/3}\right)$$

$$I = \frac{1}{10}\left(e^{-\pi/3} - e^{\pi/3}\right).$$

La DSES dans IR de $f(x) = \dfrac{x+3}{(x^2-9)(x^2+9)}$ *est de la forme :*

1. $f(x) = \dfrac{a}{x-3} + \dfrac{b}{x+3} + \dfrac{c}{x-3} + \dfrac{d}{x+3}$
2. $f(x) = \dfrac{a}{x-3} + \dfrac{bx+c}{x^2+9}$
3. $f(x) = \dfrac{ax+b}{x^2-9} + \dfrac{cx+d}{x^2+9}$
4. *Je n'en sais rien*

1%

2%

3%

4%

$$f(x) = \frac{x+3}{(x^2-9)(x^2+9)} = \frac{x+3}{(x-3)(x+3)(x^2+9)}$$

$$f(x) = \frac{1}{(x-3)\underbrace{(x^2+9)}_{(x-3j)(x+3j)}} = \frac{a}{x-3} + \frac{bx+c}{x^2+9}$$

$$f(x) = \frac{1}{(x-3)(x^2+9)} = \frac{a}{x-3} + \frac{bx+c}{x^2+9}$$

1. $a = \frac{1}{18}\ ; b = \frac{-1}{18}\ et\ c = \frac{-1}{6}$ — 1%
2. $a = -\frac{1}{18}\ ; b = \frac{1}{18}\ et\ c = \frac{-1}{6}$ — 2%
3. $a = \frac{1}{18}\ ; b = \frac{-1}{18}\ et\ c = \frac{1}{6}$ — 3%
4. $a = \frac{-1}{18}\ ; b = \frac{1}{18}\ et\ c = \frac{1}{6}$ — 4%

$$f(x) = \frac{1}{(x-3)(x^2+9)} = \frac{a}{x-3} + \frac{bx+c}{x^2+9}$$

$$a = \Big[(x-3)f(x)\Big]_{x=3} = \Big[\frac{1}{x^2+9}\Big]_{x=3} = \frac{1}{18}$$

$$b3j+c = \Big[(x^2+9)f(x)\Big]_{x=3j} = \Big[\frac{1}{x-3}\Big]_{x=3j} = \frac{1}{3j-3}$$

$$3bj+c = \frac{1}{3j-3} \times \frac{-3j-3}{-3j-3} = \frac{-3j-3}{3^2+3^2} = \frac{-3j-3}{18} = \frac{-j-1}{6}$$

$$\iff \begin{cases} 3b = -\frac{1}{6} \\ c = -\frac{1}{6} \end{cases} \iff \begin{cases} b = -\frac{1}{18} \\ c = -\frac{1}{6} = -\frac{3}{18} \end{cases}$$

$$f(x) = \frac{1}{18(x-3)} - \frac{x+3}{18(x^2+9)}$$