Corrigé TD Révisions
DS1 R204
IPP + DSES

Notes

$$I = \int_0^1 e^{3x}\cdot(x^2-3x)\,dx$$

IPP1 $\begin{cases} U = x^2-3x \Rightarrow U' = 2x-3 \\ V' = e^{3x} \Rightarrow V = \frac{1}{3}e^{3x} \end{cases}$

$$\int_a^b U\cdot V'\,dx = \left[U\cdot V\right]_a^b - \int_a^b U'\cdot V\,dx$$

$$I = \left[(x^2-3x)\,\frac{1}{3}\,e^{3x}\right]_0^1 - \int_0^1 (2x-3)\,\frac{1}{3}\,e^{3x}\,dx$$

$$I = -\frac{2}{3}e^3 - \frac{1}{3}\cdot\int_0^1 (2x-3)\,e^{3x}\,dx$$

IPP2 $\begin{cases} U = 2x-3 \Rightarrow U' = 2 \\ V' = e^{3x} \Rightarrow V = \frac{1}{3}e^{3x} \end{cases}$

$$I = -\frac{2}{3}e^3 - \frac{1}{3}\cdot\left\{\left[\frac{1}{3}(2x-3)\cdot e^{3x}\right]_0^1 - \frac{2}{3}\int_0^1 e^{3x}\,dx\right\}$$

$$\boxed{I = \frac{1}{27}\left(-11-13e^3\right)}$$

$$I = -\frac{2}{3}e^3 - \frac{1}{3}\left(-\frac{1}{3}e^3 + \frac{1}{3}\cdot 3\cdot e^0 - \frac{2}{9}\cdot\left[e^{3x}\right]_0^1\right\} = -\frac{2}{3}e^3 + \frac{1}{9}e^3 - \frac{1}{3} + \frac{2}{27}\left(e^3-1\right)$$

Notes

$$F(x) = \frac{x^2-1}{x^5-3x^4+7x^3-13x^2+12x-4} = \frac{A(x)}{B(x)}$$

① Factorisation de B dans $\mathbb{R}$:

$B(1)=0$

$B'(x) = 5x^4-12x^3+21x^2-26x+12 \Rightarrow B'(1) = 5-12+21-26+12 = 0$

$B''(x) = 20x^3-36x^2+42x-26 \Rightarrow B''(1) = 20-36+42-26 = 0$

$B^{(3)}(x) = 60x^2-72x+42 \Rightarrow B^{(3)}(1) \neq 0$

1 est donc racine triple de B. B est donc divisible par $(x-1)^3$

$(x-1)^3 = (x-1)^2(x-1) = (x^2-2x+1)(x-1) = x^3-3x^2+3x-1$

$$\begin{array}{l|l} x^5-3x^4+7x^3-13x^2+12x-4 & x^3-3x^2+3x-1 \\ \hline -(x^5-3x^4+3x^3-x^2) & x^2+4 \\ \quad 4x^3-12x^2+12x-4 & \\ \quad -(4x^3-12x^2+12x-4) & \\ \quad /0 & \end{array}$$

Notes

Donc $B(x) = (x-1)^3 \cdot (x^2+4)$ dans $\mathbb{R}$

$B(x) = (x-1)^3 \cdot (x-2j)(x+2j)$ dans $\mathbb{C}$

② $F(x) = \dfrac{x^2-1}{(x-1)^3(x^2+4)} = \dfrac{(x-1)(x+1)}{(x-1)^3(x^2+4)} = \dfrac{x+1}{(x-1)^2(x^2+4)}$ est réduite

③ $\deg A = 1 < \deg B = 4$ donc F n'a pas de partie entière

④ $F(x) = \dfrac{x+1}{(x-1)^2(x^2+4)} = \dfrac{a}{(x-1)^2} + \dfrac{b}{x-1} + \dfrac{cx+d}{x^2+4}$

$$a = \left[(x-1)^2 F(x)\right]_{x=1} = \left[\frac{x+1}{x^2+4}\right]_{x=1} = \frac{2}{5}$$

$$c \cdot 2j + d = \left[(x^2+4)\,F(x)\right]_{x=2j} = \left[\frac{x+1}{(x-1)^2}\right]_{x=2j} = \frac{2j+1}{(2j-1)^2} = \frac{2j+1}{-3-4j} \times \frac{-3+4j}{-3+4j}$$

Notes

$$c2j+d=\frac{-2j-11}{25} \iff \begin{cases} c=-\frac{1}{25} \\ d=-\frac{11}{25}\end{cases}$$

Calcul de b : $\lim\limits_{x\to+\infty} xF(x) = \lim\limits_{x\to+\infty}\left(\frac{x^2}{x^4}\right) = \lim\limits_{x\to+\infty}\left(\frac{ax}{x^2}+\frac{bx}{x}+\frac{cx^2}{x^2}\right)$

$0 = b+c \iff b=-c=\frac{1}{25}$

Donc

$$F(x)=\frac{2}{5}\,\frac{1}{(x-1)^2}+\frac{1}{25}\,\frac{1}{x-1}-\frac{1}{25}\,\frac{x+11}{x^2+4}$$

$$\int F(x)\,dx=\frac{2}{5}\int\frac{dx}{(x-1)^2}+\frac{1}{25}\int\frac{dx}{x-1}-\frac{1}{50}\cdot\int\frac{2x}{x^2+4}-\frac{11}{4\times 25}\cdot 2\int\frac{\frac{1}{2}\,dx}{\left(\frac{x}{2}\right)^2+1}$$

$$\int F(x)\,dx=-\frac{2}{5}\,\frac{1}{x-1}+\frac{1}{25}\cdot\ln|x-1|-\frac{1}{50}\ln(x^2+4)-\frac{1}{50}\arctan\left(\frac{x}{2}\right)+cte$$