Corrigé DM11

Ex1 Module et argument de :

$z_1 = 5-3j$ $\qquad |z_1| = \sqrt{5^2+3^2} = \sqrt{34}$

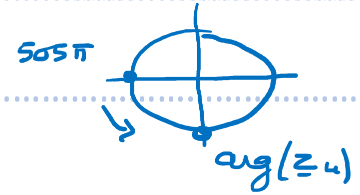
$$\arg(z_1) = \arctan(-3/5) = -\arctan(3/5)$$

$z_2 = -2+j$ $\qquad |z_2| = \sqrt{2^2+1^2} = \sqrt{5}$

$$\arg(z_2) = \arctan(-1/2) + \pi = \pi - \arctan(1/2)$$

$z_3 = \sqrt{5} - j\sqrt{2}$ $\qquad |z_3| = \sqrt{\sqrt{5}^2 + \sqrt{2}^2} = \sqrt{7}$

$$\arg(z_3) = \arctan(-\sqrt{2}/\sqrt{5}) = -\arctan(\sqrt{2}/\sqrt{5})$$



$z_4 = (1+j)^{2022}$ $\qquad |z_4| = |1+j|^{2022} = \sqrt{2}^{2022} = \left(2^{\frac{1}{2}}\right)^{2022} = 2^{1011}$

$$\arg(z_4) = 2022 \times \arg(1+j) = 2022.\arctan(1) = 2022 \times \frac{\pi}{4} = \frac{1011\pi}{2} = \frac{1010\pi}{2} + \frac{\pi}{2}$$

$$\begin{cases} = 505\pi + \frac{\pi}{2} \\ = -\frac{\pi}{2} \text{ à } 2k\pi \text{ près} \end{cases}$$

Pour z_4 : écriture exponentielle polaire algébrique

$$z_4 = 2^{1011} \cdot e^{-j\pi/2} = \left[2^{1011} ; -90°\right] = 2^{1011}\left(\underbrace{\cos\frac{\pi}{2}}_{0} - j\underbrace{\sin\frac{\pi}{2}}_{1}\right) = -2^{1011} j$$

Ex 2 Module et argument de :

Notes

$$Z_1 = \frac{3-2j}{4+3j} \qquad Z_1 = \frac{|3-2j|}{|4+3j|} = \frac{\sqrt{13}}{\sqrt{25}} = \frac{\sqrt{13}}{5}$$

$$\arg(Z_1) = \arg(3-2j) - \arg(4+3j)$$

$$\arg(Z_1) = \arctan(-2/3) - \arctan(3/4) = -\arctan\left(\frac{2}{3}\right) - \arctan\left(\frac{3}{4}\right)$$

$$Z_2 = (\sqrt{2} + j\sqrt{7})(1+j)$$

$$Z_2 = |\sqrt{2}+j\sqrt{7}| \times |1+j| = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$$

$$\arg(Z_2) = \arg(\sqrt{2}+j\sqrt{7}) + \arg(1+j) = \arctan\left(\frac{\sqrt{7}}{\sqrt{2}}\right) + \frac{\pi}{4}$$

$$Z_3 = (5+3j)^5 \qquad Z_3 = |5+3j|^5 = \sqrt{34}^5 = 34^{5/2}$$

$$Z_4 = \frac{(1+jx)^7}{(1-jx)^8} \; ; \; x \in \mathbb{R}. \quad Z_4 = \frac{|1+jx|^7}{|1-jx|^8}$$

$$Z_4 = \frac{\sqrt{1+x^2}^7}{\sqrt{1+x^2}^8} = \frac{1}{\sqrt{1+x^2}} \; ; \; \arg(Z_4) = 7.\arg(1+jx) - 8\arg(1-jx)$$

$$\arg(Z_4) = 7\arctan(x) - 8\arctan(-x) = 15\arctan(x)$$

Ex 3

* En écriture polaire, compléter :

$$[2;30°] \times [7;40°] = [14;70°]$$

$$[2;30°]^* = [2;-30°]$$

$$[3;-20°]^2 = [9;-40°]$$

$$\frac{[4;30°]}{[2;20°]} = [2;10°]$$

* Compléter :

$$60° = \frac{180°}{3} \leftrightarrow \frac{\pi}{3}$$

$$Re([2;60°]) = 2.\cos(60°) = 1$$

$$Im([2;60°]) = 2\sin(60°) = \sqrt{3}$$