Calculer les intégrales suivantes à l'aide du changement de variable proposé :

Notes

$$I_1 = \int_1^3 \frac{\sqrt{t}}{t+1}\,dt$$ en posant $x = \sqrt{t}$

Indication : après avoir fait le changement de variable, remplacer le numérateur par $x^2 = x^2$+1-1 puis séparer la fraction en deux fractions, dont vous reconnaîtrez les primitives.

Astuce

$$I_2 = \int_1^4 \frac{1-\sqrt{t}}{\sqrt{t}}\,dt$$ en posant $x = \sqrt{t}$

$$I_3 = \int_0^{\pi} \frac{\sin(t)}{1+co\ ^2(t)}\,dt$$ en posant x =cos(t)

$$I_4 = \int_1^{e} \frac{dt}{2t.ln(t)+t}$$ en posant x =ln(t)

$$I_5 = \int_0^{1/3} \frac{dt}{9t^2-6t+}$$ en posant x = 3t-1

$$I_1 = \int_1^3 \frac{\sqrt{t}}{t+1}\,dt$$ en posant $x = \sqrt{t}$

Notes

Bornes : $\begin{cases} t = 1 \\ t = 3 \end{cases} \Leftrightarrow \begin{cases} x = \sqrt{1} = 1 \\ x = \sqrt{3} \end{cases}$

Relation entre dx et dt :

$$x = \sqrt{t} \Rightarrow \frac{dx}{dt} = (\sqrt{t})' = \frac{1}{2\sqrt{t}}$$

$$\Leftrightarrow dt = 2x\,dx$$

$$I_1 = \int_1^{\sqrt{3}} \frac{x}{x^2+1} 2x\,dx = 2.\int_1^{\sqrt{3}} \frac{x^2}{x^2+1}\,dx$$

Astuces $\frac{x^2+1-1}{x^2+1} = 1 - \frac{1}{1+x^2}$

$$I_1 = 2.\int_1^{\sqrt{3}} \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= 2.\left[x - \arctan x\right]_1^{\sqrt{3}}$$

$$= 2\left(\sqrt{3} - \underbrace{\arctan\sqrt{3}}_{\frac{\pi}{3}} - (1 - \underbrace{\arctan 1}_{\frac{\pi}{4}})\right)$$

$$= 2\left(\sqrt{3} - 1 - \frac{4\pi}{4\times 3} + \frac{3\times\pi}{3\times 4}\right)$$

$$I_1 = 2\left(\sqrt{3} - 1 - \frac{\pi}{12}\right)$$

Notes

$$I_2 = \int_1^4 \frac{1-\sqrt{t}}{\sqrt{t}}\,dt$$ en posant $x = \sqrt{t}$

Bornes : $\begin{cases} t = 1 \\ t = 4 \end{cases} \Longleftrightarrow \begin{cases} x = 1 \\ x = 2 \end{cases}$

Relation dx et dt : $x = \sqrt{t}$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t}} \Longleftrightarrow dt = 2x\,dx$$

x

$$I_2 = \int_1^2 \frac{1-x}{\cancel{x}} \cdot 2\cancel{x}\,dx$$

$$= 2\int_1^2 (1-x)\,dx = 2\left[x - \frac{x^2}{2}\right]_1^2$$

$$= 2\left(2 - 2 - \left(1 - \frac{1}{2}\right)\right)$$

$I_2 = -1$

Notes $I_3 = \int_0^{\pi} \frac{\sin(t)}{1+\cos^2(t)} dt$ en posant x = cos(t)

$-dx$

$\cos 0$

Bornes : $\begin{cases} t = 0 \\ t = \pi \end{cases} \iff \begin{cases} x = 1 \\ x = -1 \end{cases}$ $\cos\pi$

dx et dt : $x = \cos t$

$\frac{dx}{dt} = -\sin t \iff \boxed{dx = -\sin t \, dt}$

$\sin t \, dt = -dx$

$I_3 = \int_1^{-1} -\frac{dx}{1+x^2} = \int_{-1}^{1} \frac{dx}{1+x^2}$ $\longrightarrow$ $-\int_a^b f(t)dt = \int_b^a f(t)dt$

$I_3 = [\text{Arctan}\, x]_{-1}^{1} = \underbrace{\text{Arctan}\, 1}_{\pi/4} - \underbrace{\text{Arctan}(-1)}_{-\pi/4}$

$I_3 = \frac{\pi}{2}$

Notes

$$I_4 = \int_1^e \frac{dt}{2t.ln(t)+t}$$ en posant $x = \ln(t) \Leftrightarrow t = e^x$

Bornes : $\begin{cases} t = 1 \\ t = e \end{cases} \Leftrightarrow \begin{cases} x = \ln 1 = 0 \\ x = \ln e = 1 \end{cases}$

dx et dt $\quad x = \ln t$

$\frac{dx}{dt} = \frac{1}{t} \Leftrightarrow \boxed{dt = e^x dx}$ $\quad t = e^x$

$$I_4 = \int_0^1 \frac{e^x dx}{2e^x . x + e^x} = \int_0^1 \frac{1 \, dx}{2x+1}$$

$$= \frac{1}{2} \int_0^1 \frac{dx}{x+\frac{1}{2}} = \frac{1}{2} \left[\ln \left| x + \frac{1}{2} \right| \right]_0^1$$

$$= \frac{1}{2} \left(\ln\left(\frac{3}{2}\right) - \ln\left(\frac{1}{2}\right) \right)$$

$$I_4 = \frac{1}{2} \cdot \ln \left(\frac{\frac{3}{\cancel{2}}}{\frac{1}{\cancel{2}}} \right) = \frac{1}{2} \ln 3$$

$\ln A - \ln B = \ln\left(\frac{A}{B}\right)$

$$I_4 = \ln \sqrt{3}$$

$\ln x^\alpha = \alpha \ln x$

$3^{1/2} = \sqrt{3}$

$$\frac{e^x}{2e^x x + e^x} = \frac{\cancel{e^x}}{\cancel{e^x}(2x+1)}$$

$$I_5 = \int_0^{1/3} \frac{\boxed{dt}}{9t^2-6t+2}$$ en posant x = 3t-1

$3t = x+1$

$t = \frac{1}{3}(x+1)$

Notes

Bornes : $\begin{cases} t=0 \\ t=1/3 \end{cases} \iff \begin{cases} x=-1 \\ x=0 \end{cases}$

dx et dt : $x = 3t-1$

$$\frac{dx}{dt} = 3 \iff dt = \frac{dx}{3}$$

$$I_5 = \int_{-1}^{0} \frac{\boxed{\frac{1}{3}\,dx}}{9\underbrace{\left(\frac{1}{3}(x+1)\right)^2}_{x^2+2x+1} - \underbrace{6\left(\frac{1}{3}(x+1)\right)}_{2x+2} + 2}$$

$$= \frac{1}{3} \cdot \int_{-1}^{0} \frac{dx}{x^2+1}$$

$$I_5 = \frac{1}{3}\left[\operatorname{Arctan} x\right]_{-1}^{0}$$

$$= \frac{1}{3}\left(\underbrace{\operatorname{Arctan} 0}_{=0} - \underbrace{\operatorname{Arctan}(-1)}_{=-\pi/4}\right)$$

$$I_5 = \frac{\pi}{12}$$

$$\left(\frac{1}{3}\cdot(x+1)\right)^2 = \frac{1}{3^2}\cdot(x+1)^2$$

$$= \frac{1}{9}\left(x^2+2x+1\right)$$