# Correction du DM7

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1) $f(x) = x^3 + 3x^2 + 3x - 1$ $\quad D_f = \mathbb{R}$

$f'(x) = 3x^2 + 6x + 3$ $\quad D_{f'} = \mathbb{R}$

2) $f(x) = x + \frac{1}{x}$ avec $x \neq 0$ $\quad D_f = \mathbb{R}^*$

$f'(x) = 1 - \frac{1}{x^2}$ $\quad D_{f'} = \mathbb{R}^*$

3) $g(t) = \frac{t+1}{t-1}$ avec $t \neq 1$ $\quad D_g = \mathbb{R} \setminus \{1\}$

$g'(t) = \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ $\quad u = t+1 \quad u' = 1$

$= \frac{t - 1 - (t+1)}{(t-1)^2}$ $\quad v = t - 1 \quad v' = 1$

$= \frac{t - 1 - t - 1}{(t-1)^2}$

$= \frac{-2}{(t-1)^2}$ $\quad D_{g'} = \mathbb{R} \setminus \{1\}$

4) $h(t) = \sqrt{t^2+1}$ $\quad D_h = \mathbb{R}$

$h'(t) = (\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$ $\quad u = t^2+1 \quad u' = 2t$

$= \dfrac{\cancel{2}t}{\cancel{2}\sqrt{t^2+1}}$

$= \dfrac{t}{\sqrt{t^2+1}}$

$= \cancel{\dfrac{t\sqrt{t^2+1}}{t^2+1}}$ $\quad D_{h'} = \mathbb{R}.$

5) $f(\omega) = \dfrac{\omega}{\sqrt{\omega^2+1}}$ $\quad D_f = \mathbb{R}$

$\mathcal{D}_l = \mathbb{R}$

$$l(\omega)=\frac{\omega}{\sqrt{\omega^2+1}}\ \forall\omega\in R\ ;\ l'(\omega)=\frac{1*\sqrt{\omega^2+1}-\frac{2\omega}{2\sqrt{\omega^2+1}}*\omega}{\omega^2+1}=\frac{\sqrt{\omega^2+1}-\frac{\omega^2}{\sqrt{\omega^2+1}}}{\omega^2+1}\ \forall\omega\in R.$$

$u = \omega \qquad u' = 1$

$v = \sqrt{\omega^2+1} = (\omega^2+1)^{1/2}$

$$\frac{\sqrt{\omega^2+1}}{1} - \frac{\omega^2}{\sqrt{\omega^2+1}}$$

$v' = \dfrac{x}{\sqrt{x^2+1}}$

$$l'(\omega) = \frac{u'v - uv'}{v^2} = \frac{\sqrt{\omega^2+1} - \omega\cdot\left(\frac{\omega}{\sqrt{\omega^2+1}}\right)}{\left(\sqrt{\omega^2+1}\right)^2}$$

$$\frac{\omega^2+1-\omega^2}{\sqrt{\omega^2+1}}$$

$$= \frac{\frac{1}{\sqrt{\omega^2+1}}}{\omega^2+1}$$

$$= \frac{1}{\sqrt{\omega^2+1}\,(\omega^2+1)}$$

$\mathcal{D}_{l'} = \mathbb{R}$

$$= \frac{1}{(\omega^2+1)^{3/2}}$$

$$X(\omega) = \left(L\omega - \frac{1}{C\omega}\right)^2 \ \forall \omega \in R^* \, ; X'(\omega) = 2 * \left(L + \frac{1}{c\omega^2}\right) * \left(L\omega - \frac{1}{C\omega}\right) \ \forall \omega \in R^*$$

$$X(\omega) = \left(L\omega - \frac{1}{C\omega}\right)^2$$

$$u(\omega) = L\omega - \frac{1}{C\omega}$$

$$X'(\omega) = 2u(\omega) \cdot u'(\omega)$$

$$u'(\omega) = L - \left(-\frac{1}{C\omega^2}\right) = L + \frac{1}{C\omega^2}$$

$$X'(\omega) = 2\left(L\omega - \frac{1}{C\omega}\right)\left(L + \frac{1}{C\omega^2}\right), \quad D_X = \mathbb{R} \setminus \{0\}$$

$$Z(\omega) = \sqrt{R^2 + L^2\omega^2}\ \forall\omega \in R\ ;\ Z'(\omega) = \frac{2L^2\omega}{2\sqrt{R^2+L^2\omega^2}} = \frac{L^2\omega}{\sqrt{R^2+L^2\omega^2}}\ \forall\omega \in R$$

8) $i(t) = I\sqrt{2}\cos(\omega t + \varphi)$ $\quad D_i = \mathbb{R}$

$(\cos(ax+b))' = -a\sin(ax+b)$

$i'(t) = I\sqrt{2} \times (-\omega) \times \sin(\omega t + \varphi)$

$= -I\sqrt{2}\,\omega\sin(\omega t + \varphi)$. $\quad D_{i'} = \mathbb{R}$

g) $f_0(C) = \frac{1}{2\pi\sqrt{LC}}$ $\qquad D_{f_0} = \mathbb{R}^+ \setminus \{0\}$

$$= \frac{1}{2\pi\sqrt{L}} \times C^{-1/2}$$

$$f'_0(C) = \frac{1}{2\pi\sqrt{L}}\left(-\frac{1}{2}\right) C^{-3/2} = -\frac{1}{4\pi\sqrt{L}}\, C^{-3/2} \qquad D_{f'_0} = \mathbb{R}$$

$$W(C) = \frac{1}{2} * \frac{Q^2}{C} = \frac{1}{2} * Q^2 * \frac{1}{C}\ \forall C \in R^*,\ w'(C) = \frac{1}{2} * Q^2 * \frac{-1}{C^2} = -\frac{Q^2}{C^2} * \frac{1}{2}\ \forall C \in R^*.$$

$$W(Q) = \frac{1}{2} * \frac{Q^2}{C}\ \forall Q \in R\,; W'(Q) = \frac{1}{2} * \frac{2Q}{C} = \frac{Q}{C}\ \forall Q \in R$$

Notes

$$V(x) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R^2} - x\right)$$ elle est definie sur IR

$$V'(x) = \frac{\sigma}{2\varepsilon_0}\left(\frac{x}{\sqrt{x^2+R^2}} - 1\right)$$ elle est definie sur IR