Exercice 5 Calculer la limite des fonctions suivantes au « point » a donné.

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1) $f(x) = \dfrac{(x^4-1)(x^2+3)}{2x(x^3-2)(x^2+2)}$ a=∞ $\lim\limits_{x\to+\infty} f(x) =$

2) $g(x) = \dfrac{(x^7-2x)(x-4)}{(x^6-2x)(x^3-x)}$ a=∞ $\lim\limits_{x\to+\infty} g(x) =$

3) $h(x) = \dfrac{x^6-3x^4+1}{(x+1)(x^4+3)}$ a=∞ $\lim\limits_{x\to+\infty} h(x) =$

4) $f(x) = \dfrac{x^2+2x-3}{x^3-1}$ a=1 $\lim\limits_{x\to1} f(x) =$

5) $f(x) = \dfrac{x^3+\sqrt{x-1}}{\sqrt{x}-2}$ a=∞ $\lim\limits_{x\to+\infty} f(x) =$

6) $f(x) = \dfrac{\sqrt{x+1}-2}{x-3}$ a=3 $\lim\limits_{x\to3} f(x) =$

7 $f(x) = \dfrac{3x^5 \cdot \sin(5x^4)}{(3x^2+2x)^4 \cdot \ln(1+2x)}$

$\lim\limits_{x\to0} f(x) =$

1) $f(x)=\dfrac{(x^4-1)(x^2+3)}{2x(x^3-2)(x^2+2)}$ $\quad$ a=∞

* FI $\dfrac{\text{"}\infty\text{"}}{\infty}$ $\quad$ $f(x) \underset{+\infty}{\sim} \dfrac{x^4 \times x^2}{2x \times x^3 \times x^2} = \dfrac{x}{2x} = \dfrac{1}{2}$

Donc $\lim\limits_{x \to +\infty} f(x) = \dfrac{1}{2}$.

Autre rédaction : $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{x^4 \times x^2}{2x \times x^3 \times x^2} = \lim\limits_{x \to +\infty} \dfrac{x^6}{2x^6} = \dfrac{1}{2}$.

* $\lim\limits_{x \to -\infty} f(x) = \dfrac{1}{2}$ car $f(x) \underset{-\infty}{\sim} \dfrac{1}{2}$.

$\dfrac{3}{8 \times -0{,}0001} = \dfrac{-3}{8} 10000..$

* $\lim\limits_{x \to 0} f(x)$ $\quad$ $f(x) \underset{0}{\sim} \dfrac{-1 \times 3}{2x \times (-2) \times (2)} = \dfrac{3}{8x}$ donc $\lim\limits_{x \to 0} f(x)$ $\begin{cases} \lim\limits_{x \to 0^+} f(x) = +\infty \\ \lim\limits_{x \to 0^-} f(x) = -\infty \end{cases}$

2) $g(x) = \dfrac{(x^7 - 2x)(x - 4)}{(x^6 - 2x)(x^3 - x)}$ $a = \infty$

3) $h(x) = \dfrac{x^6 - 3x^4 + 1}{(x+1)(x^4+3)}$ $a = \infty$

4) $f(x) = \dfrac{x^2+2x-3}{x^3-1}$ $a=1$

$\to g(x) \Rightarrow g'(x) = 2x+2$

$\to h(x) \Rightarrow h'(x) = 3x^2$

$$\lim_{x\to 1} f(x) = \text{"}\frac{0}{0}\text{"}$$

$$f(x) \underset{x\to 1}{\sim} \frac{g(1)+(x-1)\cdot g'(1)}{h(1)+(x-1)\cdot h'(1)} = \frac{0+(x-1)\times 4}{0+(x-1)\times 3} = \frac{4}{3}$$

donc $\lim_{x\to 1} f(x) = \frac{4}{3}$.

$$5)\ f(x)=\frac{x^3+\sqrt{x-1}}{\underbrace{\sqrt{x}}_{x^{1/2}}-2.x^0}\quad a=\infty$$

$\sqrt{x-1} = (x-1)^{1/2}$

FI "$\frac{\infty}{\infty}$"

$$f(x)\underset{+\infty}{\sim}\frac{x^3}{x^{1/2}}=x^{2,5}=x^2.\sqrt{x}$$

donc $\lim\limits_{x\to+\infty} f(x)=+\infty.$

6) $f(x) = \dfrac{\sqrt{x+1} - 2}{x - 3}$ $a=3$

expression conjuguée

7) $f(x) = \dfrac{3x^5 \cdot \sin(5x^4)}{(3x^2+2x)^4 \cdot \ln(1+2x)}$ $\qquad a = 0$