Tp 11.

exercice 1 :

$$f(x) = \frac{7-2x}{(x+1)(x-2)} = \frac{a}{(x+1)} + \frac{b}{(x-2)}$$

$$a = \left[(x+1)\cdot f(x)\right]_{x=-1} = \left[\frac{7-2x}{(x-2)}\right]_{x=-1} = \frac{7+2}{-3} = \frac{9}{-3} = -3$$

$$b = \left[(x-2)\cdot f(x)\right]_{x=2} = \left[\frac{7-2x}{(x+1)}\right]_{x=2} = \frac{7-4}{3} = 1$$

$$f(x) = \frac{-3}{(x+1)} + \frac{1}{(x-2)}$$

$$\Leftrightarrow \int f(x)\,dx = -3\ln|x+1| + \ln|x-2| + C$$

exercice 2 :

$$g(x) = \frac{2x+3}{x^3-3x^2-4x+12} = \frac{A(x)}{B(x)}$$

$B(2) = 8 - 12 - 8 + 12 = 0$

$B(-2) = 0$

$B'(x) = 3x^2 - 3 \times 2x - 4 = 3x^2 - 6x - 4$

$B'(2) = 12 - 12 - 4 \neq 0$

$B'(-2) \neq 0$

B est donc divisible par $(x-2)(x+2) = x^2 + 2x - 2x - 4$ $\Leftrightarrow x^2 - 4$

$$\begin{array}{l|l} x^3 - 3x^2 - 4x + 12 & x^2 - 4 \\ \underline{-(x^3 - 4x)} & \\ -3x^2 + 8x + 12 & x - 3 \\ \underline{-(-3x^2 + 12)} & \\ 8x & \end{array}$$

$B(x) = \dots (x-3)(x-2)(x+2)$

$g(x) = \hat{g}(x) = \dfrac{a}{(x-2)} + \dfrac{b}{(x+2)} + \dfrac{c}{(x-3)}$

$$a = \left[(x-2)\cdot g(x)\right]_{x=2} = \left[\frac{2x+3}{(x+2)(x-3)}\right]_{x=2} = \frac{7}{-4}$$

$$b = \left[(x+2)\cdot g(x)\right]_{x=-2} = \left[\frac{2x+3}{(x-2)(x-3)}\right]_{x=-2} = \frac{-1}{20}$$

$$c = \left[(x-3)\cdot g(x)\right]_{x=3} = \left[\frac{2x+3}{(x+2)(x-2)}\right]_{x=3} = \frac{9}{5}$$

$$\text{donc } g(x) = \frac{\frac{7}{-4}}{(x-2)} - \frac{\frac{1}{20}}{(x+2)} + \frac{9/5}{(x-3)}$$

$$\text{donc } \int g(x)\,dx = \frac{-7}{4}\ln|x-2| - \frac{1}{20}\ln|x+2| + \frac{9}{5}\ln|x-3| + k$$

exercice 3 :

$$h(x) = \frac{x+3}{(x-1)^2(x^2+1)} = \frac{A(x)}{B(x)} = \frac{a}{(x-1)^2} + \frac{b}{x-1} + \frac{cx+d}{x^2+1}$$

1 double

j et $-j$

$(x-j)(x+j)$

$$a = \left[(x-1)^2 \cdot h(x)\right]_{x=1} = \left[\frac{x+3}{(x^2+1)}\right] = \frac{4}{2} = 2.$$

$$b = \left[(x-1)\cdot h(x)\right]_{x=1} = \left[\frac{x+3}{x-1}\right]$$

$$cj+d \left[(x^2+1)\cdot h(x)\right]_{x=j} = \left[\frac{j+3}{(j-1)^2}\right]_{x=j} = \frac{j+3}{-2j} = \frac{j}{-2j} + \frac{3}{-2j} \times \frac{j}{j}$$

$$cj+d = \frac{1}{-2} + \frac{3}{2}j \quad \text{donc} \quad c = \frac{3}{2} \quad d = -1/2$$

$$h(0) = 2 - b - \frac{1}{2} = 3$$

$$b = 2 - \frac{1}{2} - 3 = -1 - \frac{1}{2} = \frac{-10}{2} - \frac{11}{2} = \frac{-11}{2}$$

$$b = -\frac{3}{2}.$$

$$h(x) = \frac{2}{(x-1)^2} - \frac{3/2}{(x-1)} + \frac{3/2\,x - 1/2}{(x^2-1)}$$

$$\int h(x)\,dx = \frac{-2}{x-1} - \frac{3}{2}\ln|(x-1)| + \frac{3}{4}\ln|(x^2+1)| - \frac{-1}{2}\arctan(x)$$

exercice 4 : TD 11

1 - $\frac{t-1}{(t^2-1)(t+2)} = \frac{t-1}{(t^2+1)(t-1)+2)} = \frac{t-1}{t-1} \times \frac{1}{(t+1)(t+2)}$

$= 1 \times \frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)(t+2)} = \frac{a}{t+1} + \frac{b}{t+2}$.

$a = \left[(t+1)\, r(t)\right]_{x=-1} = \left[\frac{1}{(t+2)}\right]_{x=-1} = 1$

$b = \left[(t+2)\, r(t)\right]_{x=-2} = \left[\frac{1}{(t+1)}\right]_{x=-2} = -1$

$r(t) = \frac{1}{(t+1)} - \frac{1}{(t+2)}$

2 - $\int_{-4}^{-3} r(t)\, dt = \left[\ln|t+2| - \ln|t+1|\right]_{-4}^{-3} = \ln|1| - \ln(-2) + \ln(-2) + \ln(-3)$

$= \ln 3/4$