Tp 12

exercice 1 :

$$AP = \int_{-\pi/6}^{\pi/6} \cos(5t)\,dt \overset{2}{=} \left[\frac{1}{5}\sin(5t)\,dt\right]_{0}^{\pi/6}$$

$$= \frac{1}{5}\cancel{\sin\left(5\times\frac{\pi}{6}\right) - \frac{1}{5}\sin\left(5\times\left(-\frac{\pi}{6}\right)\right)}$$

$$= 2\left(\frac{1}{5}\sin\left(5\times\frac{\pi}{6}\right) - \frac{1}{5}\sin(0)\right)$$

$$= 2\times\frac{1}{10} = \frac{1}{5} + cte$$

$$AK = \int_{0}^{\pi} \sin(3t).\cos^{8}(3t)\,dt = -\frac{1}{3}\int_{0}^{\pi} -3\sin(3t).\cos^{8}(3t)\,dt$$

$$\int u'u^{\alpha}\,dx = \frac{u^{\alpha+1}}{\alpha+1} = \frac{\cos(3t)^{9}}{9}$$

$$= -\frac{1}{3}\left[\frac{\cos(3t)^9}{9}\right]_0^{\pi} = -\frac{1}{3}\left(\frac{\cos^9(3t)}{9} - \frac{\cos^9(0)}{9}\right) = \frac{2}{27}$$

$$\star\ P = \int_0^{\pi/3} \sin\left(2t + \frac{\pi}{3}\right) dt = \left[\frac{-1}{2}\cos\left(2t + \frac{\pi}{3}\right)\right]_0^{\pi/3}$$

$$= \frac{-1}{2}\cos\left(\frac{2\pi}{3} + \frac{\pi}{3}\right) - \left(\frac{-1}{2}\cos\left(0 + \frac{\pi}{3}\right)\right)$$

$$= -\frac{1}{2}\cos(3\pi) + \frac{1}{2}\cos\left(\frac{\pi}{3}\right) = \frac{3}{4}$$

$$\star\ H(x) = \int \frac{x+2}{x^2 - 2x - 3}\, dx$$

$$\Delta = b^2 - 4ac = (-2)^2 - 4 \times 1 \times (-3) = 4 + 12 = 16$$

$$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{2+4}{2} = 3$$

$$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{2-4}{2} = \frac{-2}{2} = -1$$

Les racines sont $(x-3)(x+1)$

Donc $f(x) = \int \dfrac{x+2}{(x-3)(x+1)}\,dx$

On cherche les constantes a et b $\dfrac{a}{(x-3)} + \dfrac{b}{(x+1)}$

$$a = \Big[(x-3)f(x)\Big]_{x=3} = \left[\frac{x+2}{x+1}\right]_{x=3} = \frac{5}{4}$$

$$b = \Big[(x+1)f(x)\Big]_{x=-1} = \left[\frac{x+2}{x-3}\right]_{x=-1} = \frac{1}{-4}$$

donc $f(x)\ \dfrac{\frac{5}{4}}{(x-3)} - \dfrac{\frac{1}{4}}{(x+1)}$

les primitives sont $F(x) = \dfrac{5}{4}\ln|x-3| - \dfrac{1}{4}\ln|x+1| + cte$

$$*\ Q = \int_{\pi/6}^{\pi/3} \tan x \, dx = -\int_{\pi/6}^{\pi/3} \frac{\sin x}{\cos x} \, dx = -\Big[\ln|\cos x|\Big]_{\pi/6}^{\pi/3}$$

$$= -\ln\left|\cos\left(\frac{\pi}{3}\right)\right| + \ln\left|\cos\left(\frac{\pi}{6}\right)\right| = -\ln\left|\frac{1}{2}\right| + \ln\left|\frac{\sqrt{3}}{2}\right|$$

$$= \ln\left(\frac{\sqrt{3}}{2}\right) - \ln\left(\frac{1}{2}\right) = \ln\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right| = \ln|\sqrt{3}|$$

$$*\ N(x) = \int \frac{x}{x^3 - x^2 + x - 1} \, dx$$

$N(1) = 1 - 1 + 1 - 1 = 0$

$N'(x) = 3x^2 - 2x + 1$

$N'(1) = 3 - 2 + 1$ donc 1 est une racine simple de $N(x)$

$$\begin{array}{r|l} x^3 - x^2 + x - 1 & x - 1 \\ \hline -(x^3 - x^2) & x^2 + 1 \\ x - 1 & \\ -(x - 1) & \\ 0 & \end{array}$$

On vérifie : $(x^2+1)(x-1)$

$\Leftrightarrow x^3 - x^2 + x - 1$

On factorise (x^2+1)

$(x+j)(x-j)$

Donc, le dénominateur : $(x-1)(x+j)(x-j)$ dans $\mathbb{C}$

$(x-1)(x^2+1)$ dans $\mathbb{R}$

On cherche les constantes a et b

$$\frac{a}{(x-1)} + \frac{cx+b}{(x^2+1)}$$

$$a = \Big[(x-1).f(x)\Big]_{x=1} = \frac{x}{(x^2+1)} = \frac{1}{2}$$

$$cj+b = \Big[(x^2+1).f(x)\Big]_{x=-j} = \Big[\frac{x}{(x-1)}\Big]_{x=-j} = \frac{j}{j-1} \times \frac{j+1}{j+1}$$

$$= \frac{j-1}{-2} = \frac{-j}{2} + \frac{1}{2}$$

$$c = \frac{-1}{2}$$

$$b = \frac{1}{2}$$

donc $f(x) = \dfrac{\frac{1}{2}}{(x-1)} - \dfrac{\frac{1}{2}x + \frac{1}{2}}{(x^2+1)}$

On cherche les primitives de $f(x) : \frac{1}{2} \ln|x+1| - \frac{1}{2 \times 2} \int \frac{x}{x^2+1}\, dx$

$$- \frac{1}{2} \int \frac{1}{x^2-1} \cdot dx$$

$$= -\frac{1}{2} \ln|x+1| - \frac{1}{2} \arctan(x) - \frac{1}{4} \ln|x^2+1| + cte$$