Brouillon

$K(t) = \frac{2}{2}\int \frac{2(2t^3+2t)}{2\sqrt{t^4+2.t^2+1}}dt =$

$\int \frac{U'}{2\sqrt{U}}dt = \sqrt{U}$ $\quad U = t^4 + 2t^2 + 1 \Rightarrow U' = 4t^3 + 4t = 2(2t^3 + 2t)$

donc $K(t) = \sqrt{t^4 + 2t^2 + 1} + cte$

$J(t) = \frac{1}{-4}\int 3.\sin(4t).\cos^7(4t)dt =$

$\int U'U^7 dt = \frac{U^8}{8} + cte$ $\quad U = \cos(4t) \Rightarrow U' = -4\sin(4t)$

$J(t) = -\frac{3}{4}.\int -4\sin(4t).\cos^7(4t)\,dt = -\frac{3}{4}\frac{\cos^8(4t)}{8} + cte$

2) Calculer les intégrales suivantes :

Brouillon

$L = \int_{-2}^{-1} \frac{3}{t}\,dt = 3 \int_{-2}^{-1} \frac{dt}{t} = 3 \times \left[\ln|t|\right]_{-2}^{-1} = 3\left(\ln|-1| - \ln|-2|\right)$

$L = 3(\ln 1 - \ln 2) = -3\ln 2$

$M = \int_{-\pi}^{\pi} \underbrace{x^3}_{\text{impaire}} . \overbrace{\cos(7x)}^{\text{paire}}\,dx$ donc $x^3 . \cos(7x)$ est impaire sur $[-\pi ; \pi]$, centré en 0

donc $M = 0$.

$N = \int_0^1 t.\sqrt{e^{-t^2}}dt = \int_0^1 t.(e^{-t^2})^{1/2} dt = -\int_0^1 -t.e^{-\frac{t^2}{2}} dt$

$\int u'.e^u \, dt = e^u + cte \qquad u = -\frac{t^2}{2} \Rightarrow u' = -\frac{2t}{2} = -t$

$N = -\int_0^1 -te^{-t^2/2} dt = -\left[e^{-t^2/2}\right]_0^1 = -(e^{-1/2} - e^0) = 1 - e^{-1/2} = 1 - \frac{1}{\sqrt{e}}.$