Tp 13 :

A LPES → cos, sin
↙ arcos, arcsin ↓ ln ↘ polynômes ↘ exp

exercice 2 p 19 :

$$* I = \int_0^{\pi} (2x+1)\sin(3x)\,dx$$

$$\int_a^b u.v'\,dt = [uv]_a^b - \int_a^b u'.v\,dt$$

$u = (2x+1)$ $\quad u' = 2$

$v' = \sin(3x)$ $\quad v = -\cos(3x)$

$$I = \left[(2x+1)\times(-\cos(3x))\right]_0^{\pi} - \int_0^{\pi} -\frac{2}{3}\cos(3x)$$

$$= (-(2\pi+1)\times\frac{1}{3}\cos(3\pi)) - \frac{1}{3} + \frac{2}{3}\int_0^{\pi}\cos(3x)$$

$$\cancel{= -2\pi - 2 + 2\left[\cos(3x)\right]_0^{\pi}}$$

$$\cancel{= -2\pi - 2 + 2(\cos(3\pi) - \cos(0))}$$

$$= \frac{2\pi+1}{3} + \frac{1}{3} + \frac{2}{3}\left[\frac{1}{3}\sin(3x)\right]_0^{\pi}$$

$$= \frac{2\pi+1}{3} + \frac{1}{3} + \frac{2}{3}\left(\left(\frac{1}{3}\sin 3\pi\right) - 0\right)$$

$$= \frac{2\pi+1}{3} + \frac{1}{3} + 0 = \frac{2\pi+1}{3} + \frac{1}{3} = \frac{2\pi+2}{3}$$

$$* N(x) = \int_0^1 \arctan x\,dx.$$

$u = \arctan x \quad u' = \frac{1}{1+x^2}$
$v' = 1 \quad v = x$

$$N = \left[\arctan x \times x\right]_0^1 - \int_0^1 \frac{1}{1+x^2} \times x \, dx$$

$$= (\arctan 1 \times 1) - (\arctan 0 \times 0) - \int_0^1 \frac{x}{1+x^2} \, dx \quad \frac{u'}{u}$$

$$= \frac{\pi}{4} - \frac{1}{2} \int_0^1 \frac{2x}{1+x^2} \, dx$$

$$= \frac{\pi}{4} - \frac{1}{2} \left[\ln|1+x^2|\right]_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}\left(\ln(2) - \ln(1)\right) = \frac{\pi}{4} - \frac{\ln(2)}{2}.$$

$$* \; K = \int_0^{\pi/2} e^{-x} \cos x \, dx$$

$u = e^{-x} \quad u' = -e^{-x}$
$v' = \cos x \quad v = \sin x$

$$K = \left[e^{-x} \times \sin x\right]_0^{\pi/2} - \int_0^{\pi/2} -e^{-x} \cdot \sin x \, dx$$

$$= e^{-\pi/2} + \int_0^{\pi/2} e^{-x} \cdot \sin \, dx.$$

$$= e^{-\pi/2} + \left[e^{-x} \times (-\cos x)\right]_0^{\pi/2}$$

$$= e^{-\pi/2} + \left[-e^{-x} \times \cos x\right]_0^{\pi/2} - \int_0^{\pi/2} +e^{-x} \times \cos x \, dx$$

$= e^{-\pi/2} + 1 - K \iff 2K = e^{-\pi/2} + 1$

$K = \dfrac{e^{-\pi/2} + 1}{2}.$

exercice 4p 18 :

$H(x) = \int x^2 e^{3x}\, dx$

$u = x^2 \qquad u' = 2x$

$v' = e^{3x} \qquad v = \frac{1}{3} e^{3x}$

$= \left[x^2 \times \frac{1}{3} e^{3x} \right] - \int 2x \times \frac{1}{3} e^{3x}\, dx$

$\longleftarrow$ IPP

$= \left[x^2 \times \frac{1}{3} e^{3x} \right] - \frac{2}{3} \int x \times e^{3x}\, dx \qquad u = x \quad u' = 1 \quad v' = e^{3x} \quad v = \frac{1}{3} e^{3x}$

$= \left[x^2 \times \frac{1}{3} e^{3x} \right] - \frac{2}{3} \left(\left[\frac{x}{3} \times e^{3x} \right] - \int \frac{1}{3} e^{3x}\, dx \right)$

$= \left[x^2 \times \frac{1}{3} e^{3x} \right] - \frac{2}{3} \left(\left[\frac{x}{3} e^{3x} \right] - \frac{1}{3} \left[\frac{1}{3} e^{3x} \right] \right)$

$= x^2 \times \frac{1}{3} e^{3x} - \frac{2x e^{3x}}{9} + \frac{2}{9} \left[\frac{1}{3} e^{3x} \right]$

$= \frac{x^2}{3} e^{3x} - \frac{2x e^{3x}}{9} + \frac{2}{27} e^{3x}$

$= e^{3x} \left(\frac{x^2}{3}{}_{\times 9} + \frac{2}{27} - \frac{2x}{9}{}_{\times 3} \right)$

$= \frac{e^{3x}}{27} \left(9x^2 + 2 - 6x \right) + cte$

$* \; V(t) = \int (t^3 + 2t + 3) \cdot \ln(t)\, dt$

$u = \ln t \qquad u' = \frac{1}{t}$

$v' = (t^3 + 2t + 3) \qquad v = \frac{t^4}{4} + \frac{2t^2}{2} + 3t$

$= \left[\ln t \times \left(\frac{t^4}{4} + t^2 + 3t \right) \right] - \int \frac{1}{t} \left(\frac{t^4}{4} + t^2 + 3t \right) dt$

$= \left[\ln t \times \left(\frac{t^4}{4} + t^2 + 3t \right) \right] - \int \frac{t^3}{4} + t + 3 \; dt$

$= \left[\ln t \times \left(\frac{t^4}{4} + t^2 + 3t \right) \right] - \left[\frac{1}{4} \cdot \frac{t^4}{4} + \frac{t^2}{2} + 3t \right]$

$= \ln t \times \left(\frac{t^4}{4} + t^2 + 3t \right) - \left(\frac{t^4}{16} + \frac{t^2}{2} + 3t \right) + cte$