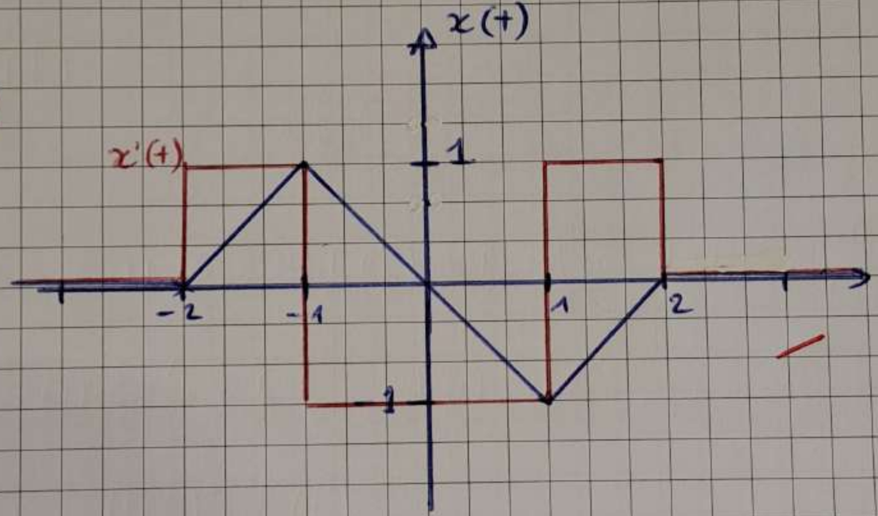
Ex1 : (4)

1) $x(t) = -\text{rect}(t + \frac{3}{2}) + 2\ \text{tri}\left(\frac{t-2}{2}\right)$

2) $X(f) = -e^{-2\pi j f \times -\frac{3}{2}} \times \text{sinc}(f) + 2 \times 2\ e^{-2\pi j f \times 2} \times \text{sinc}^2(2f)$

$X(f) = -e^{3\pi j f}\ \text{sinc}(f) + 4\ e^{-4j\pi f}\ \text{sinc}^2(2f)$



Ex2 : 1) (8)

2) Méthode 1 :

$x'(t) = \text{rect}(t + \frac{3}{2}) - \text{rect}(\frac{t}{2}) + \text{rect}(t - \frac{3}{2})$

TF

$X'(f) = e^{-2\pi j f \times -\frac{3}{2}} \times \text{sinc}(f) - 2\ \text{sinc}(2f) + e^{-2\pi j f \times \frac{3}{2}} \times \text{sinc}(f)$

$= e^{3\pi j f} \times \text{sinc}(f) - 2\ \text{sinc}(2f) + e^{-3j\pi f}\ \text{sinc}(f)$

$= \left(e^{3\pi j f} + e^{-3j\pi f}\right) \text{sinc}(f) - 2\ \text{sinc}(2f)$

$X'(f) = 2\cos(3\pi f)\ \text{sinc}(f) - 2\ \text{sinc}(2f)$

Or $X'(f) = 2\pi j f \times X(f)$

d'où $X(f) = \dfrac{X'(f)}{2\pi j f} = \dfrac{\cos(3\pi f) \times \text{sinc}(f) - \text{sinc}(2f)}{\pi j f}$

$$X(f) = \frac{-j\cos(3\pi f)\,\text{sinc}(f) + j\,\text{sinc}(2f)}{\pi f}$$

3) Méthode 2 : $x(t) = \text{tri}(t+1) - \text{tri}(t-1)$

$\xrightarrow{TF}$ $X(f) = e^{-2\pi j f \times -1}\,\text{sinc}^2(f) - e^{-2\pi j f \times 1} \times \text{sinc}^2(f)$

$= e^{2\pi j f}\,\text{sinc}^2(f) - e^{-2\pi j f}\,\text{sinc}^2(f)$

$= \text{sinc}^2(f)\left(e^{2\pi j f} - e^{-2\pi j f}\right)$

$X(f) = 2j\sin(2\pi f)\,\text{sinc}^2(f)$

Ex 3 : 5

1) $5\frac{ds(t)}{dt} + 2s(t) = 4e(t)$

$\xrightarrow{TF}$ $5 \times 2\pi j f \times S(f) + 2S(f) = 4E(f)$

$\rightarrow (10j\pi f + 2)\,S(f) = 4E(f)$

$\Leftrightarrow \frac{S(f)}{E(f)} = \frac{2}{5j\pi f + 1} \Rightarrow H(f).$

2) on a $H(f) = \frac{2}{5j\pi f + 1} \xrightarrow{TF^{-1}} h(t) = \frac{4}{5} \times e^{-\frac{2}{5}t}\,\mathcal{U}(t)$

$$= \frac{2}{\frac{5}{2}\left(2j\pi f + \frac{2}{5}\right)}$$

$$= 2 \times \frac{2}{5}\left(\frac{1}{2j\pi f + \frac{2}{5}}\right)$$

$$H(f) = \frac{4}{5} \times \frac{1}{2j\pi f + \underbrace{\frac{2}{5}}_{=a}}$$

TF^{-1}

3) si $e(t) = \delta(t)$ alors $E(f) = 1$ d'où $H(f) = \frac{S(f)}{E(f)} = S(f)$

$\Leftrightarrow h(t) = s(t) = \frac{4}{5} \times e^{-\frac{2}{5}t}\,\mathcal{U}(t).$

4) $S(f) = H(f) \times E(f)$

$\xrightarrow{TF^{-1}} s(t) = h(t) * e(t) = \int_{-\infty}^{+\infty} \mathcal{U}(u) \times \frac{4}{5}e^{-\frac{2}{5}(t-u)}\,\mathcal{U}(t-u)\,du$

convolution

$= \frac{4}{5}\int_0^{+\infty} 1 \times e^{-\frac{2}{5}(t-u)}\,\mathcal{U}(t-u)\,du$

Changement de variable :

soit $v = t - u$

$\begin{cases} u \to +\infty \\ u \to 0 \end{cases} \Leftrightarrow \begin{cases} v \to -\infty \\ v \to t \end{cases}$ et $\frac{dv}{du} = -1$ alors $du = -dv$

$$s(t) = \frac{4}{5} \int_{-\infty}^{t} e^{-\frac{2}{5}v}\, \mathcal{U}(v)\, dv$$

1er cas : si $t < 0$

alors $s(t) = 0$ car $\mathcal{U}(v) = 0$

2ème cas : $t \geqslant 0$

$$s(t) = \frac{4}{5}\left[\int_{-\infty}^{0} 0\, dv + \int_{0}^{t} e^{-\frac{2}{5}v}\, \mathcal{U}(v)\, dv\right]$$

$$= \frac{4}{5}\int_{0}^{t} e^{-\frac{2}{5}v}\, dv$$

$$= \frac{4}{5} \times -\frac{5}{2}\left[e^{-2/5 v}\right]_0^t = -2\left(e^{-\frac{2}{5}t} - 1\right) = 2 - 2e^{-\frac{2}{5}t}$$

Conclusion :

$$s(t) = \begin{cases} 2 - 2e^{-\frac{2}{5}t} & \text{si } t \geqslant 0 \\ 0 & \text{sinon.} \end{cases}$$

dans le cas où e est un échelon unit

TB

$$s(t) = -2\left(e^{-2/5 t} - 1\right)$$

<u>Conclusion</u> $$s(t) = 2\left(1 - e^{-2/5 t}\right) . U(t)$$

<u>Ex4</u> $$(f * g)(t) = (g * f)(t)$$

$$= \int_{-\infty}^{+\infty} g(u) . f(t-u)\, du$$

$$= \int_{-1/2}^{1/2} 3 . 5 . \cos(3t - 3u)\, du$$

$$= 15 \times \left[\frac{\sin(3t-3u)}{-3} \right]_{-1/2}^{1/2}$$

$\boxed{6}$

$$(f*g)(t) = -5\left(\sin(3t-3/2) - \sin(3t+3/2)\right)$$

$$= 5\left(\sin 3t \cdot \cos 3/2 + \sin 3/2 \cos 3t - \sin 3t \cos 3/2 + \right.$$
$$\left. + \sin 3/2 \cos 3t\right)$$

$$= 5 \times 2 \times \sin 3/2 \cdot \cos 3t$$

$$(f*g)(t) = 10 \cdot \sin \frac{3}{2} \cdot \cos(3t)$$