TP2

Ex2

$$3s'(t) + 2s(t) = 2e(t)$$

① $6j\pi f S(f) + 2S(f) = 2E(f)$

$$S(f)\cdot(6j\pi f + 2) = 2E(f)$$

$$H(f) = \frac{S(f)}{E(f)} = \frac{2}{6j\pi f + 2}$$

$$H(f) = \frac{1}{3j\pi f + 1}$$

② $h(t) = T_F^{-1}(H(f))$

$$H(f) = \frac{1}{3j\pi f + 1} \times \frac{\frac{2}{3}}{\frac{2}{3}}$$

$$H(f) = \frac{2}{3} \frac{1}{\frac{2}{3} + 2j\pi f}$$

$$h(t) = \frac{2}{3} \cdot e^{-\frac{2}{3}t} \cdot U(t)$$

③ Réponse impulsionnelle :
lorsque $e(t) = \delta(t)$ alors

$$E(f) = 1$$

$$S(f) = H(f) \times E(f)$$

$$S(f) = H(f)$$

$s(t) = h(t) = \frac{2}{3} e^{-\frac{2}{3}t} . U(t)$ est
donc la réponse impulsionnelle

④ $e(t) = \text{rect}(t)$

$$S(f) = H(f) \times E(f)$$

$$s(t) = h(t) * e(t)$$

↑ produit de convolution :

$$s(t) = \frac{2}{3} e^{-\frac{2}{3}t} . U(t) * \text{rect}(t)$$

$$s(t) = \frac{2}{3} . \int_{-\infty}^{+\infty} \text{rect}(u) \, e^{-\frac{2}{3}(t-u)} \, U(t-u) \, du$$

$$s(t) = \frac{2}{3} \int_{-1/2}^{1/2} e^{-\frac{2}{3}(t-u)} \cdot U(t-u)\, du$$

On pose $\theta = t - u$

u varie entre $-1/2 \Leftrightarrow \theta = t + 1/2$

et $1/2 \Leftrightarrow \theta = t - 1/2$

$d\theta = -du$

$$s(t) = \frac{2}{3} \int_{t+1/2}^{t-1/2} - e^{-\frac{2}{3}\theta} U(\theta)\, d\theta$$

$$s(t) = \frac{2}{3} \cdot \int_{t-1/2}^{t+1/2} e^{-\frac{2}{3}\theta} \cdot U(\theta)\, d\theta$$

<u>1er cas</u> $t + 1/2 < 0 \Leftrightarrow t < -1/2$

$s(t) = 0$

<u>2e cas</u> $t - 1/2 < 0 < t + 1/2$

$\Leftrightarrow -\frac{1}{2} < t < \frac{1}{2}$

$$s(t) = \frac{2}{3} \cdot \int_0^{t+1/2} e^{-\frac{2}{3}v} \, dv$$

$$s(t) = \left[-e^{-\frac{2}{3}v} \right]_0^{t+1/2}$$

$$s(t) = 1 - e^{-\frac{2}{3}(t+1/2)}$$

3^e^ Cas $t - 1/2 > 0 \iff t > 1/2$

$$s(t) = \frac{2}{3} \cdot \int_{t-1/2}^{t+1/2} e^{-\frac{2}{3}v} \, dv$$

$$s(t) = \left[-e^{-\frac{2}{3}v} \right]_{t-1/2}^{t+1/2}$$

$$s(t) = e^{-\frac{2}{3}(t-1/2)} - e^{-\frac{2}{3}(t+1/2)}$$

$$= e^{-2/3 t} \cdot \left(e^{1/3} - e^{-1/3} \right)$$

$$s(t) = 2 e^{-\frac{2}{3}t} \, \mathrm{sh}(1/3)$$

la réponse est donc :

$$s(t) = \begin{cases} 0 \text{ si } t \leq -1/2 \\ 1 - e^{-\frac{2}{3}(t+\frac{1}{2})} \text{ si } -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 2 e^{-\frac{2}{3}t} \operatorname{sh}(1/3) \quad t \geq \frac{1}{2} \end{cases}$$

Ex 4

$$(f*g)(t) = \int_{-\infty}^{+\infty} e^{3u} \operatorname{rect}(u) e^{-4(t-u)} \cdot U(t-u) \, du$$

$$(f*g)(t) = \int_{-1/2}^{1/2} e^{3u} \cdot e^{-4(t-u)} \cdot U(t-u) \, du$$

On pose $v = t - u$ $\quad dv = -du$

$u = -1/2 \Leftrightarrow v = t + 1/2$

$u = 1/2 \Leftrightarrow v = t - 1/2$

$$(f*g)(t) = \int_{t+1/2}^{t-1/2} -e^{3(t-v)} \cdot e^{-4v} \cdot U(v) \, dv$$

$$(f*g)(t) = \int_{t-1/2}^{t+1/2} e^{3t} \cdot e^{-7\theta} \cdot U(\theta)\, d\theta$$

$$(f*g)(t) = e^{3t} \int_{t-1/2}^{t+1/2} e^{-7\theta} \cdot U(\theta)\, d\theta$$

1^e Cas $t + 1/2 < 0 \Leftrightarrow t < -1/2$

$$(f*g)(t) = 0$$

2^e Cas : $t - 1/2 < 0 < t + 1/2 \Leftrightarrow -1/2 < t < \frac{1}{2}$

$$(f*g)(t) = e^{3t} \int_{0}^{t+1/2} e^{-7\theta}\, d\theta$$

$$= e^{3t} \left[\frac{e^{-7\theta}}{-7} \right]_0^{t+1/2}$$

$$(f*g)(t) = \frac{e^{3t}}{7} \left(1 - e^{-7(t+1/2)} \right)$$

3^e Cas $t - 1/2 > 0 \Leftrightarrow t > 1/2$

$$(f*g)(t) = e^{3t} \cdot \int_{t-1/2}^{t+1/2} e^{-7\theta}\, d\theta$$

$$(f*g)(t) = e^{3t} \cdot \left[\frac{e^{-7u}}{-7} \right]_{t-7/2}^{t+7/2}$$

$$(f*g)(t) = \frac{e^{3t}}{7} \cdot \left(e^{-7(t-7/2)} - e^{-7(t+7/2)} \right)$$

$$(f*g)(t) = \frac{e^{-4t}}{7} \left(e^{7/2} - e^{-7/2} \right)$$

$$(f*g)(t) = \frac{2e^{-4t}}{7} \, sh(7/2)$$